Integration of Rational Functions Exam Questions MS

Q1, (OCR 4724, Jun 2006, Q3)

(i)	$\frac{A}{x}+\frac{B}{3-x}$ & c-u rule or $A(3-x)+Bx\equiv 3-2x$	**M1**		Correct format + suitable method
	$\frac{1}{x}$	**A1**		seen in (i) or (ii)
	$-\frac{1}{3-x}$	**A1**	**3**	ditto; $\frac{1}{x}-\frac{1}{3-x}$ scores 3 immediately
(ii)	$\int\frac{1}{x}(\mathrm{d}x)=\ln x$ or $\ln\lvert x\rvert$	**B1**		
	$\int\frac{1}{3-x}(\mathrm{d}x)=-\ln(3-x)$ or $-\ln\lvert 3-x\rvert$	**B1**		Check sign carefully; do not allow $\ln(x-3)$
	Correct method idea of substitution of limits	**M1**		Dep on an attempt at integrating
	$\ln 2\ (+\ln 1\ -\ln 1)\ -\ln 2\ =0$	**A1**	**4**	Clearly seen; WWW **AG**
	Alternative Method			
	If ignoring PFs, $\ln x(3-x)$ immediately	**B2**		$\ln x(x-3)\to 0$
	As before	**M1,A1**	**(4)**	
(iii)	Suitable statement or clear implication e.g. Equal amounts (of area) above and below (axis) or graph crosses axis or there's a root (Be lenient)	**B1**	**1**	

Q2, (OCR 4724, Jan 2007, Q6)

(i) $2x+1=/\equiv A(x-3)+B$	M1		
$A=2$	A1		
$B=7$	A/B 1	**3**	Cover-up rule acceptable for B1
(ii) $\int\frac{1}{x-3}(\mathrm{d}x)=\ln(x-3)$ or $\ln\lvert x-3\rvert$	B1		Accept A or $\frac{1}{A}$ as a multiplier
$\int\frac{1}{(x-3)^2}(\mathrm{d}x)=-\frac{1}{x-3}$	B1		Accept B or $\frac{1}{B}$ as a multiplier
$6+2\ln 7$ Follow-through $\frac{6}{7}B+A\ln 7$	√B2	**4**	

Q3, (OCR 4724, Jun 2009, Q6)

(i)	$4x \equiv A(x-3)^2 + B(x-3)(x-5) + C(x-5)$	M1		
	$A = 5$	A1		'cover-up' rule, award B1
	$B = -5$	A1		
	$C = -6$	A1	**4**	'cover-up' rule, award B1
	Cands adopting other alg. manip. may be awarded M1 for a full satis method + 3 @ A1			

(ii)	$\int \frac{A}{x-5}\,dx = A\ln(5-x)$ or $A\ln\lvert 5-x\rvert$ or $A\ln\lvert x-5\rvert$	√B1		but <u>not</u> $A\ln(x-5)$
	$\int \frac{B}{x-3}\,dx = B\ln(3-x)$ or $B\ln\lvert 3-x\rvert$ or $B\ln\lvert x-3\rvert$	√B1		but <u>not</u> $B\ln(x-3)$
	<u>If candidate is awarded B0,B0,</u> then award **SR** √ B1 for $A\ln(x-5)$ **and** $B\ln(x-3)$			
	$\int \frac{C}{(x-3)^2}\,dx = -\frac{C}{x-3}$	√B1		
	$5\ln\frac{3}{4} + 5\ln 2$ aef, isw √ $A\ln\frac{3}{4} - B\ln 2$ √	B1		Allow if **SR** B1 awarded
	-3 √ $\frac{1}{2}C$	√B1	**5**	
	[Mark at earliest correct stage & isw; no ln 1]		**9**	

Q4, (OCR 4724, Jun 2007, Q7)

(i) Leading term in quotient = $2x$	B1		
<u>Suff evidence</u> of division or identity process	M1		
Quotient = $2x+3$	A1		Stated or in relevant position in division
Remainder = x	A1	4	Accept $\frac{x}{x^2+4}$ as remainder
(ii) their quotient + $\frac{\text{their remainder}}{x^2+4}$	√B1	1	$2x+3+\frac{x}{x^2+4}$
(iii) <u>Working with their expression in part (ii)</u>			
their $Ax + B$ integrated as $\frac{1}{2}Ax^2 + Bx$	√B1		
their $\frac{Cx}{x^2+4}$ integrated as $k\ln(x^2+4)$	M1		Ignore any integration of $\frac{D}{x^2+4}$
$k = \frac{1}{2}C$	√A1		
Limits used correctly throughout	M1		
$14 + \frac{1}{2}\ln\frac{13}{5}$	A1	5	logs need not be combined.
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