Exponential and Logarithmic Equations Exam Questions

Q1 (OCR 4722, Jun 2009, Q3)

Use logarithms to solve the equation $7^x = 2^{x+1}$, giving the value of x correct to 3 significant figures. **[5]**

Q2, (OCR 4722, Jan 2010, Q8)

(a) Use logarithms to solve the equation $5^{3w-1} = 4^{250}$, giving the value of w correct to 3 significant figures. **[5]**

(b) Given that $\log_x(5y + 1) - \log_x 3 = 4$, express y in terms of x. **[4]**

Q3, (OCR 4722, Jun 2008, Q8)

(i) Sketch the curve $y = 2 \times 3^x$, stating the coordinates of any intersections with the axes. [3]

(ii) The curve $y = 2 \times 3^x$ intersects the curve $y = 8^x$ at the point P. Show that the x-coordinate of P may be written as

$$\frac{1}{3 - \log_2 3}.$$ [5]

Q4, (OCR 4722, Jan 2010, Q9)

(i) Sketch the curve $y = 6 \times 5^x$, stating the coordinates of any points of intersection with the axes. **[3]**

(ii) The point P on the curve $y = 9^x$ has y-coordinate equal to 150. Use logarithms to find the x-coordinate of P, correct to 3 significant figures. **[3]**

(iii) The curves $y = 6 \times 5^x$ and $y = 9^x$ intersect at the point Q. Show that the x-coordinate of Q can be written as $x = \dfrac{1 + \log_3 2}{2 - \log_3 5}$. **[5]**

Q5, (OCR 4722 Jan 2009, Q8)

(a) Given that $\log_a x = p$ and $\log_a y = q$, express the following in terms of p and q.

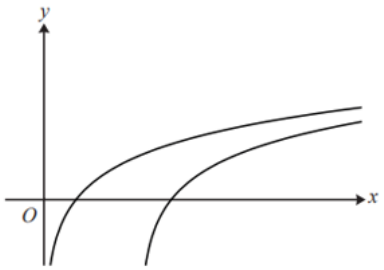
(i) $\log_a(xy)$ **[1]**

(ii) $\log_a\left(\dfrac{a^2x^3}{y}\right)$ **[3]**

(b) **(i)** Express $\log_{10}(x^2 - 10) - \log_{10} x$ as a single logarithm. **[1]**

(ii) Hence solve the equation $\log_{10}(x^2 - 10) - \log_{10} x = 2\log_{10} 3$. **[5]**

Q6, (OCR 4722, Jan 2013, Q8)



The diagram shows the curves $y = \log_2 x$ and $y = \log_2 (x - 3)$.

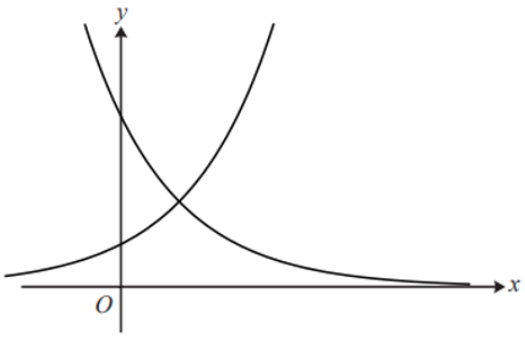
(i) Describe the geometrical transformation that transforms the curve $y = \log_2 x$ to the curve $y = \log_2 (x - 3)$. **[2]**

(ii) The curve $y = \log_2 x$ passes through the point $(a, 3)$. State the value of a. **[1]**

(iii) The curve $y = \log_2 (x - 3)$ passes through the point $(b, 1.8)$. Find the value of b, giving your answer correct to 3 significant figures. **[2]**

(iv) The point P lies on $y = \log_2 x$ and has an x-coordinate of c. The point Q lies on $y = \log_2 (x - 3)$ and also has an x-coordinate of c. Given that the distance PQ is 4 units find the exact value of c. **[4]**

Q7, (OCR 4722, Jun 2013, Q8)



The diagram shows the curves $y = a^x$ and $y = 4b^x$.

(i) **(a)** State the coordinates of the point of intersection of $y = a^x$ with the y-axis. **[1]**

(b) State the coordinates of the point of intersection of $y = 4b^x$ with the y-axis. **[1]**

(c) State a possible value for a and a possible value for b. **[2]**

(ii) It is now given that $ab = 2$. Show that the x-coordinate of the point of intersection of $y = a^x$ and $y = 4b^x$ can be written as

$$x = \frac{2}{2\log_2 a - 1}.$$

[5]

Q8, (OCR 4723, Jan 2006, Q6)

(a)

t	0	10	20
X	275	440	

The quantity X is increasing exponentially with respect to time t. The table above shows values of X for different values of t. Find the value of X when $t = 20$. [3]

(b) The quantity Y is decreasing exponentially with respect to time t where

$$Y = 80e^{-0.02t}.$$

(i) Find the value of t for which $Y = 20$, giving your answer correct to 2 significant figures. [3]

(ii) Find by differentiation the rate at which Y is decreasing when $t = 30$, giving your answer correct to 2 significant figures. [3]

Q9, (OCR 4723, Jun 2008, Q7)

It is claimed that the number of plants of a certain species in a particular locality is doubling every 9 years. The number of plants now is 42. The number of plants is treated as a continuous variable and is denoted by N. The number of years from now is denoted by t.

(i) Two equivalent expressions giving N in terms of t are

$$N = A \times 2^{kt} \quad \text{and} \quad N = Ae^{mt}.$$

Determine the value of each of the constants A, k and m. [4]

(ii) Find the value of t for which $N = 100$, giving your answer correct to 3 significant figures. [2]

(iii) Find the rate at which the number of plants will be increasing at a time 35 years from now. [3]

Q10, (OCR 4723, Jan 2009, Q5)

The mass, M grams, of a certain substance is increasing exponentially so that, at time t hours, the mass is given by

$$M = 40e^{kt},$$

where k is a constant. The following table shows certain values of t and M.

t	0	21	63
M		80	

(i) In either order,

(a) find the values missing from the table, **[3]**

(b) determine the value of k. **[2]**

(ii) Find the rate at which the mass is increasing when $t = 21$. **[3]**

Q11, (OCR 4723, Jan 2012, Q7)

(i) Substance A is decaying exponentially and its mass is recorded at regular intervals. At time t years, the mass, M grams, of substance A is given by

$$M = 40e^{-0.132t}.$$

(a) Find the time taken for the mass of substance A to decrease to 25% of its value when $t = 0$. **[3]**

(b) Find the rate at which the mass of substance A is decreasing when $t = 5$. **[3]**

(ii) Substance B is also decaying exponentially. Initially its mass was 40 grams and, two years later, its mass is 31.4 grams. Find the mass of substance B after a further year. **[3]**

Q12, (OCR 4723, Jun 2014, Q5)

(a) The mass, M grams, of a substance at time t years is given by

$$M = 58e^{-0.33t}.$$

Find the rate at which the mass is decreasing at the instant when $t = 4$. Give your answer correct to 2 significant figures. **[3]**

(b) The mass of a second substance is increasing exponentially. The initial mass is 42.0 grams and, 6 years later, the mass is 51.8 grams. Find the mass at a time 24 years after the initial value. **[4]**

Q13, (OCR 4723, Jun 2016, Q3)

The mass of a substance is decreasing exponentially. Its mass is m grams at time t years. The following table shows certain values of t and m.

t	0	5	10	25
m	200	160		

(i) Find the values missing from the table. **[2]**

(ii) Determine the value of t, correct to the nearest integer, for which the mass is 50 grams. **[4]**
