Exponential and Logarithmic Equations Exam Questions MS

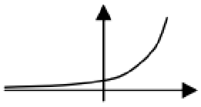
Q1, (OCR 4722, Jun 2009, Q3)

$\log 7^x = \log 2^{x+1}$	M1		Introduce logarithms throughout, or equiv with base 7 or 2
$x\log 7 = (x+1)\log 2$	M1		Drop power on at least one side
	A1		Obtain correct linear equation (allow with no brackets)
$x(\log 7 - \log 2) = \log 2$	M1		**Either** expand bracket and attempt to gather x terms, **or** deal correctly with algebraic fraction
$x = 0.553$	A1	5	Obtain $x = 0.55$, or rounding to this, with no errors seen
		5	

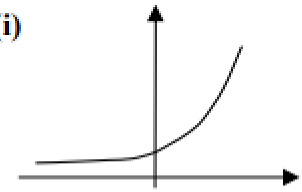
Q2, (OCR 4722, Jun 2010, Q8)

a	$\log 5^{3w-1} = \log 4^{250}$	**M1***		Introduce logarithms throughout
	$(3w-1)\log 5 = 250\log 4$	**M1***		Use $\log a^b = b\log a$ at least once
	$3w - 1 = \frac{250\log 4}{\log 5}$	**A1**		Obtain $(3w-1)\log 5 = 250\log 4$ or equiv
	$w = 72.1$	**M1d***		Attempt solution of linear equation
		A1	5	Obtain 72.1, or better
b	$\log_x \frac{5y+1}{3} = 4$	**M1**		Use $\log a - \log b = \log {}^a/_b$ or equiv
	$\frac{5y+1}{3} = x^4$	**M1**		Use $f(y) = x^4$ as inverse of $\log_x f(y) = 4$
	$5y + 1 = 3x^4$	**M1**		Attempt to make y the subject of $f(y) = x^4$
	$y = \frac{3x^4 - 1}{5}$	**A1**	4	Obtain $y = \frac{3x^4 - 1}{5}$, or equiv
			9	

Q3, (OCR 4722, Jun 2008, Q8)

(i) 	**M1**	Attempt sketch of exponential graph (1st quad) - if seen in 2nd quad must be approx correct
	A1	Correct graph in both quadrants
	B1	State or imply (0, 2) only
	3	
(ii) $8^x = 2 \times 3^x$		
$\log_2 8^x = \log_2 (2 \times 3^x)$	**M1**	Form equation in x and take logs (to any consistent base, or no base) – could use $\log_8$
$x\log_2 8 = \log_2 2 + x\log_2 3$	**M1**	Use $\log a^b = b \log a$
	M1	Use $\log ab = \log a + \log b$,or equiv with $\log {}^a/_b$
$3x = 1 + x\log_2 3$	**M1**	Use $\log_2 8 = 3$
$x(3 - \log_2 3) = 1$, hence $x = \dfrac{1}{3 - \log_2 3}$ **A.G.**	**A1**	Show given answer correctly
OR $8^x = 2 \times 3^x$		
$2^{3x} = 2 \times 3^x$	**M1**	Use $8^x = 2^{3x}$
$2^{(3x-1)} = 3^x$	**M1**	Attempt to rearrange equation to $2^k = 3^x$
$\log_2 2^{(3x-1)} = \log_2 3^x$	**M1**	Take logs (to any base)
$(3x - 1)\log_2 2 = x \log_2 3$	**M1**	Use $\log a^b = b \log a$
$x(3 - \log_2 3) = 1$, hence $x = \dfrac{1}{3 - \log_2 3}$ **A.G.**	**A1**	Show given answer correctly
	5	

Q4, (OCR 4722, Jan 2010, Q9)

(i) 	M1		Reasonable graph in both quadrants
	A1		Correct graph in both quadrants
	B1	3	State or imply (0, 6)
(ii) $9^x = 150$	M1		Introduce logarithms throughout, or equiv with $\log_9$
$x \log 9 = \log 150$	M1		Use $\log a^b = b \log a$ and attempt correct method to find x
$x = 2.28$	A1	3	Obtain $x = 2.28$
(iii) $6 \times 5^x = 9^x$	M1		Form eqn in x and take logs throughout (any base)
$\log_3 (6 \times 5^x) = \log_3 9^x$	M1		Use $\log a^b = b \log a$ correctly on $\log 5^x$ or $\log 9^x$ or legitimate combination of these two
$\log_3 6 + x\log_3 5 = x\log_3 9$	M1		Use $\log ab = \log a + \log b$ correctly on log (6×5^x) or log 6
$\log_3 3 + \log_3 2 + x\log_3 5 = 2x$	M1		Use $\log_3 9 = 2$ or equiv (need base 3 throughout that line)
$x(2 - \log_3 5) = 1 + \log_3 2$			
$x = \dfrac{1 + \log_3 2}{2 - \log_3 5}$ **A.G.**	A1	5	Obtain $x = \dfrac{1 + \log_3 2}{2 - \log_3 5}$ convincingly (inc base 3 throughout)
		11	

Q5, (OCR 4722 Jan 2009, Q8)

(a)(i) $\log_a xy = p + q$	B1	**1**	State $p + q$ cwo
(ii) $\log_a \left(\frac{a^2x^3}{y}\right) = 2 + 3p - q$	M1		Use $\log a^b = b \log a$ correctly at least once
	M1		Use $\log \frac{a}{b} = \log a - \log b$ correctly
	A1	**3**	Obtain $2 + 3p - q$
(b)(i) $\log_{10} \frac{x^2-10}{x}$	B1	**1**	State $\log_{10} \frac{x^2-10}{x}$ (with or without base 10)
(ii) $\log_{10} \frac{x^2-10}{x} = \log_{10} 9$	B1		State or imply that $2\log_{10} 3 = \log_{10} 3^2$
$\frac{x^2-10}{x} = 9$	M1		Attempt correct method to remove logs
$x^2 - 9x - 10 = 0$	A1		Obtain correct $x^2 - 9x - 10 = 0$ aef, no fractions
$(x - 10)(x + 1) = 0$	M1		Attempt to solve three term quadratic
$x = 10$	A1	**5**	Obtain $x = 10$ only
		10	

Q6, (OCR 4722, Jan 2013, Q8)

(i)		Translation of 3 units in positive *x*-direction	B1 B1	State translation State or imply 3 units in positive *x*-direction
(ii)		$a = 8$	B1 **[1]**	State 8
(iii)		$b - 3 = 2^{1.8}$ $b = 6.48$	B1 B1 **[2]**	State or imply $b - 3 = 2^{1.8}$ Obtain 6.48, or better
(iv)		$\log_2 c - \log_2(c - 3) = 4$ $\log_2 {}^{c}/_{c-3} = 4$ ${}^{c}/_{c-3} = 2^4$ $c = 16c - 48$ $c = {}^{48}/_{15} = {}^{16}/_{5}$	M1 M1 A1 A1 **[4]**	Equate difference in *y*-coordinates to ± 4 Use $\log a - \log b = \log {}^{a}/_{b}$ Obtain ${}^{c}/_{c-3} = 2^4$ Obtain ${}^{16}/_{5}$ oe

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Q7, (OCR 4722, Jun 2013, Q8)

(i)	**(a)**	(0, 1)	B1 **[1]**	State (0, 1)
	(b)	(0, 4)	B1 **[1]**	State (0, 4)
	(c)	State a possible value for *a* State a possible value for *b*	B1 B1 **[2]**	Must satisfy $a > 1$ Must satisfy $0 < b < 1$

(ii)	$\log_2 a^x = \log_2(4b^x)$	M1	Equate a^x and $4b^x$ and introduce logarithms at some stage
	$\log_2 a^x = \log_2 4 + \log_2 b^x$	M1	Use $\log ab = \log a + \log b$ correctly
	$x\log_2 a = \log_2 4 + x\log_2 b$	M1	Use $\log a^b = b \log a$ correctly at least once
	$x\log_2 a = \log_2 4 + x\log_2(^2/_a)$	B1	Use $b = {}^2/_a$ to produce a correct equation in a and x only
	$x\log_2 a = 2 + x\log_2 2 - x\log_2 a$ $x(2\log_2 a - 1) = 2$ $x = \dfrac{2}{2\log_2 a - 1}$ **AG**	A1	Obtain given relationship with no wrong working
		[5]	

Q8, (OCR 4723, Jan 2006, Q6)

(a)	Either:	State proportion $\frac{440}{275}$	**B1**	
		Attempt calculation involving proportion	**M1**	[involving multn and X value]
		Obtain 704	**A1 3**	
	Or:	Use formula of form $275e^{kt}$ or $275a^t$	**M1**	[or equiv]
		Obtain $k = 0.047$ or $a = \sqrt[10]{1.6}$	**A1**	[or equiv]
		Obtain 704	**A1 (3)**	[allow ±0.5]
(b)(i)		Attempt correct process involving logarithm	**M1**	[or equiv including systematic trial and improvement attempt]
		Obtain $\ln \frac{20}{80} = -0.02t$	**A1**	[or equiv]
		Obtain 69	**A1 3**	[or greater accuracy; scheme for T&I: M1A2]
(ii)		Differentiate to obtain $k\,e^{-0.02t}$	**M1**	[any constant k different from 80]
		Obtain $-1.6e^{-0.02t}$ (or $1.6e^{-0.02t}$)	**A1**	[or unsimplified equiv]
		Obtain 0.88	**A1 3**	[or greater accuracy; allow –0.88]

Q9, (OCR 4723, Jun 2008, Q7)

(i)	State $A = 42$	**B1**	
	State $k = \frac{1}{9}$	**B1**	or 0.11 or greater accuracy
	Attempt correct process for finding m	**M1**	involving logarithms or equiv
	Obtain $\frac{1}{9}\ln 2$ or 0.077	**A1**	or 0.08 or greater accuracy
		4	
(ii)	Attempt solution for t using either formula	**M1**	using correct process (log'ms or T&I or …)
	Obtain 11.3	**A1**	or greater accuracy; allow 11.3 ± 0.1
		2	
(iii)	Differentiate to obtain form Be^{mt}	**M1**	where B is different from A
	Obtain $3.235e^{0.077t}$	**A1**√	or equiv; following their A and m
	Obtain 47.9	**A1**	allow 48 or greater accuracy
		3	

Q10, (OCR 4723, Jan 2009, Q5)

(i)	State 40	B1		
	Attempt value of k using 21 and 80	M1		or equiv
	Obtain $40e^{21k} = 80$ and hence 0.033	A1		or equiv such as $\frac{1}{21}\ln 2$
	Attempt value of M for $t = 63$	M1		using established formula or using exponential property
	Obtain 320	A1	**5**	or value rounding to this
(ii)	Differentiate to obtain $ce^{0.033t}$ or $40ke^{kt}$	M1		any constant c different from 40
	Obtain $40 \times 0.033e^{0.033t}$	A1√		following their value of k
	Obtain 2.64	A1	**3**	allow 2.6 or 2.64 ± 0.01 or greater accuracy (2.64056…)
			8	

Q11, (OCR 4723, Jan 2012, Q7)

(i)	(a)	State or imply $e^{-0.132t} = 0.25$ Attempt solution of eqn of form $e^{-0.132t} = k$ Obtain 10.5	B1 M1 A1 **[3]**	or equiv such as $40e^{-0.132t} = 10$ using sound process; implied by correct ans; allow trial and improvement attempt or greater accuracy
(i)	(b)	Differentiate to obtain $ke^{-0.132t}$ Obtain $5.28e^{-0.132t}$ or $-5.28e^{-0.132t}$ Substitute 5 to obtain 2.73 or –2.73	M1 A1 A1 **[3]**	where k is a constant not equal to 40 (allow even if process looks like integration) or (unsimplified) equiv accept 2.7 or –2.7 or greater accuracy; allow 2.73 or –2.73 whatever it is claimed to be
(ii)		<u>EITHER</u> Attempt to solve $40e^{2\lambda} = 31.4$ or $40e^{-2\lambda} = 31.4$ Obtain or imply $40e^{-0.121t}$ Substitute 3 to obtain 27.8 <u>OR</u> Attempt calculation involving multiplication of power of $\frac{31.4}{40}$ Obtain $31.4 \times (\frac{31.4}{40})^{0.5}$ or $40 \times (\frac{31.4}{40})^{1.5}$ Obtain 27.8	M1 A1 A1 **[3]** M1 A1 A1	using sound process; method implied by correct formula for mass of B obtained or greater accuracy (–0.12103..) or $0.5\ln 0.785$ accept 28 or greater accuracy accept 28 or greater accuracy

Q12, (OCR 4723, Jun 2014, Q5)

(a)		Differentiate to produce $ke^{-0.33t}$	M1	where constant k is different from 58
		Obtain $-19.14e^{-0.33t}$ or $19.14e^{-0.33t}$	A1	or unsimplified equiv
		Obtain -5.1 or 5.1	A1	whatever they claim value represents; accept 5.11 but not greater accuracy
			[3]	
(b)		Either:		
		State or imply formula $42e^{kt}$ or $42a^t$	B1	$42e^{-kt}$, $42e^{-kx}$, etc. also acceptable
		Attempt to find k from $42e^{6k} = 51.8$ or a from $42a^6 = 51.8$	M1	using sound process involving logarithms at least as far as $6k = \ldots$ or $a = \ldots$
		Obtain $k = 0.035$ or $a = 1.0356$	A1	or greater accuracy 0.03495… or exact equiv $\frac{1}{6}\ln\frac{37}{30}$
		Substitute 24 to obtain value between 97.1 and 97.3 inclusive	A1	allow greater accuracy than 3 s.f.
		Or:		
		Use ratio $\frac{51.8}{42}$ in calculation	B1	
		Attempt calculation of form $42 \times r^n$	M1	
		Obtain $42 \times (\frac{51.8}{42})^4$ or $51.8 \times (\frac{51.8}{42})^3$	A1	
		Obtain value between 97.1 and 97.3 inclusive	A1	allow greater accuracy than 3 s.f.
			[4]	

Q13, (OCR 4723, Jun 2016, Q3)

i		Obtain 128 for value corresponding to 10	B1	Allow any value rounding to 128
		Obtain 65.5 for value corresponding to 25	B1	Allow any value rounding to 65 or 66; whether obtained using powers of 0.8 or by use of formula
			[2]	
ii		Attempt to find formula for m of form $200e^{kt}$ or $200 \times r^{\lambda t}$	M1	Whether attempted in part (i) or (ii)
		Obtain $200e^{(0.2\ln 0.8)t}$ or $200e^{-0.0446t}$ or $200 \times 0.8^{0.2t}$ or 200×0.956^t	A1	Or equiv
		Show correct process for solving equation of form $200e^{kt} = 50$ or $200r^{\lambda t} = 50$	M1	
		Obtain 31	A1	Or greater accuracy rounding to 31; ignore any units given; second M1 is implied by correct answer
			[4]	