

Connected Rates Of Change (From OCR 4723)

Q1, (Jan 2007, Q4)

- (i) Obtain derivative of form $k(4t + 9)^{-\frac{1}{2}}$ M1 any constant k
 Obtain correct $2(4t + 9)^{-\frac{1}{2}}$ A1 or (unsimplified) equiv
 Obtain derivative of form $k e^{\frac{1}{2}x+1}$ M1 any constant k different from 6
 Obtain correct $3e^{\frac{1}{2}x+1}$ A1 4 or equiv
- (ii) Either: Form product of two derivatives M1 numerical or algebraic
 Substitute for t and x in product M1 using $t = 4$ and calculated value of x
 Obtain 39.7 A1 3 allow ± 0.1 ; allow greater accuracy
- Or: Obtain $k(4t + 9)^n e^{\frac{1}{2}(4t+9)^{\frac{1}{2}} + 1}$ M1 differentiating $y = 6e^{\frac{1}{2}(4t+9)^{\frac{1}{2}} + 1}$
 Obtain correct $6(4t + 9)^{-\frac{1}{2}} e^{\frac{1}{2}(4t+9)^{\frac{1}{2}} + 1}$ A1 or equiv
 Substitute $t = 4$ to obtain 39.7 A1 (3) allow ± 0.1 ; allow greater accuracy

Q2, (Jan 2008, Q4)

- (i) Obtain derivative of form $kh^5(h^6 + 16)^n$ M1 any constant k ; any $n < \frac{1}{2}$; allow if
 Obtain correct $3h^5(h^6 + 16)^{-\frac{1}{2}}$ A1 or (unsimplified) equiv; no -4 now
 Substitute to obtain 10.7 A1 3 or greater accuracy or exact equiv
- (ii) Attempt multn or divn using 8 and answer from (i) M1 M1
 Attempt 8 divided by answer from (i) A1√ 3 or greater accuracy; allow 0.75 ± 0.01 ;
 Obtain 0.75 following their answer from (i)

Q3, (Jan 2010, Q7a)

- Either: State or imply either $\frac{dA}{dr} = 2\pi r$ or $\frac{dA}{dt} = 250$ B1 or both
 Attempt manipulation of derivatives
 to find $\frac{dr}{dt}$ M1 using multiplication / division
 Obtain correct $\frac{250}{2\pi r}$ A1 or equiv
 Obtain 1.6 A1 4 or equiv; allow greater accuracy
- Or: Attempt to express r in terms of t M1 using $A = 250t$
 Obtain $r = \sqrt{\frac{250t}{\pi}}$ A1 or equiv
 Differentiate $kt^{\frac{1}{2}}$ to produce $\frac{1}{2}kt^{-\frac{1}{2}}$ M1 any constant k
 Substitute $t = 7.6$ to obtain 1.6 A1 (4) allow greater accuracy

Q4, (Jan 2011, Q3)

<u>Either:</u> State or imply $8\pi r$ as derivative	B1	or equiv
Attempt to connect 12 and their derivative	M1	numerical or algebraic; using multiplication or division
Obtain $8\pi \times 150 \times 12$ and hence 45000 or 14400π or 14000π	A1	3 or equiv; or greater accuracy (45239); condone absence of units or use of wrong units
<u>Or:</u> Use $r = 12t$ to show $S = 576\pi t^2$	B1	
Attempt $\frac{dS}{dt}$ and substitute for t	M1	
Obtain $1152\pi \times \frac{150}{12}$ and hence 45000 or 14400π or 14000π	A1	(3) or equiv; or greater accuracy (45239); condone absence of units or use of wrong units

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Q5, (Jun 2012, Q6)

(i)		<p>Attempt use of chain rule</p> <p>Obtain $9h(3h^2 + 4)^{\frac{1}{2}}$</p> <p>Substitute 0.6 in attempt at first derivative</p> <p>Obtain 12.17</p>	<p>*M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>to obtain derivative of form $kh(3h^2 + 4)^n$, any non-zero constants k and n</p> <p>condone retention of -8</p> <p>or (unsimplified) equiv; no -8 here</p> <p>dep *M; condone retention of -8 here; implied by their value following wrong derivative if no working seen</p> <p>or greater accuracy</p>	
(ii)		<p>State or imply that $\frac{dh}{dr} = -0.015$ or 0.015</p> <p>Carry out multiplication of $(\pm)0.015$ and answer from part (i)</p> <p>Obtain 0.18 or -0.18 (whatever this value is claimed to be)</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>implied by use in calculation with part (i) answer</p> <p>or greater accuracy; condone absence or misuse of negative signs throughout; ignore units; allow for answer rounding to 0.18 following slight inaccuracy due to use of 12.18 or 12.2 or ...</p>	

Q6, (Jun 2013, Q3)

(i)	<p>Use α (possibly implicitly) to state that radius of 'base' is $\frac{1}{2}x$</p> <p>Substitute into formula to obtain $\frac{1}{3}\pi(\frac{1}{2}x)^2x$ or $\frac{1}{3}\pi\frac{1}{4}x^2x$ and obtain $\frac{1}{12}\pi x^3$</p>	<p>*B1</p> <p>B1</p> <p>[2]</p>	<p>or to obtain equiv such as $2r = x$ or $\frac{r}{x} = \frac{1}{2}$ or $\frac{x}{r} = 2$</p> <p>dep *B; AG; necessary detail needed</p> <p>Note: comparing formulae $\frac{1}{3}\pi r^2h$ and $\frac{1}{12}\pi x^3$ to 'deduce' is B0B0</p>
(ii)	<p>Differentiate to obtain $\frac{1}{4}\pi x^2$ or equiv</p> <p>Attempt division involving 14 and their value of derivative when $x = 8$</p> <p>Obtain 0.28</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>whatever they call it</p> <p>ie $14 \div \text{deriv}$ or $\text{deriv} \div 14$ with $x = 8$</p> <p>allow 0.279 but not greater accuracy</p> <p>Alternatives:</p> <p>1. $14t = \frac{1}{12}\pi x^3$ Obtain $\frac{dt}{dx} = \frac{1}{56}\pi x^2$ B1 Sub 8 and invert M1 Ans A1</p> <p>2. $x^3 = \frac{168t}{\pi}$ Obtain $3x^2 \frac{dx}{dt} = \frac{168}{\pi}$ B1 Sub 8 M1 Ans A1</p>

Q7, (Jun 2015, Q3)

<p>Differentiate to obtain $kh^n(2 + \sqrt{h})^5$</p> <p>Obtain $9h^{-\frac{1}{2}}(2 + \sqrt{h})^5$ or unsimplified equiv</p> <p>Divide 150 by their derivative, algebraic or numerical</p> <p>Substitute $h = 1.4$ and evaluate</p> <p>Obtain 0.06 or 0.060 or 0.0603</p>	<p>M1</p> <p>A1</p> <p>*M1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>Any non-zero constants k, n; condone presence of -192 here</p> <p>Without -192 now</p> <p>Using any recognisable attempt at first derivative</p> <p>Dep *M; assume appropriate substitution if calculation goes wrong</p> <p>But not greater accuracy in final answer; units not needed unless change made to metres and/or hours</p>	
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Q8, (Jun 2018, Q6a)

Differentiate V to obtain form $k(5 + 2x)^2$	M1	M0 if -250 retained in derivative
Obtain correct $12(5 + 2x)^2$ or (unsimplified) equiv	A1	If V expanded, $300 + 240x + 48x^2$ earns B2, and award B1 if this has only one error
Divide 15 by their attempt at $\frac{dV}{dx}$ with $x = 1.6$	M1	Not dependent on first M but attempt at differentiation must have occurred
Obtain 0.019 or 0.0186	A1	Accept greater accuracy; no need for units unless adjustment made in which case units must be clearly shown (e.g. 0.00031 ms^{-1})
[4]		