

Compound Angle Formulae Exam Questions (From OCR 4723)

Q1, (Jan 2009, Q9)

(i) By first expanding $\cos(2\theta + \theta)$, prove that

$$\cos 3\theta \equiv 4 \cos^3 \theta - 3 \cos \theta. \quad [4]$$

(ii) Hence prove that

$$\cos 6\theta \equiv 32 \cos^6 \theta - 48 \cos^4 \theta + 18 \cos^2 \theta - 1. \quad [3]$$

(iii) Show that the only solutions of the equation

$$1 + \cos 6\theta = 18 \cos^2 \theta$$

are odd multiples of 90° . [5]

Q2, (Jan 2010, Q9)

The value of $\tan 10^\circ$ is denoted by p . Find, in terms of p , the value of

(i) $\tan 55^\circ$, [3]

(ii) $\tan 5^\circ$, [4]

(iii) $\tan \theta$, where θ satisfies the equation $3 \sin(\theta + 10^\circ) = 7 \cos(\theta - 10^\circ)$. [5]

Q3, (Jun 2011, Q9)

(i) Prove that $\frac{\sin(\theta - \alpha) + 3 \sin \theta + \sin(\theta + \alpha)}{\cos(\theta - \alpha) + 3 \cos \theta + \cos(\theta + \alpha)} \equiv \tan \theta$ for all values of α . [5]

(ii) Find the exact value of $\frac{4 \sin 149^\circ + 12 \sin 150^\circ + 4 \sin 151^\circ}{3 \cos 149^\circ + 9 \cos 150^\circ + 3 \cos 151^\circ}$. [3]

(iii) It is given that k is a positive constant. Solve, for $0^\circ < \theta < 60^\circ$ and in terms of k , the equation

$$\frac{\sin(6\theta - 15^\circ) + 3 \sin 6\theta + \sin(6\theta + 15^\circ)}{\cos(6\theta - 15^\circ) + 3 \cos 6\theta + \cos(6\theta + 15^\circ)} = k. \quad [4]$$

Q4, (Jan 2013, Q9)

(i) Prove that

$$\cos^2(\theta + 45^\circ) - \frac{1}{2}(\cos 2\theta - \sin 2\theta) \equiv \sin^2 \theta. \quad [4]$$

(ii) Hence solve the equation

$$6 \cos^2\left(\frac{1}{2}\theta + 45^\circ\right) - 3(\cos \theta - \sin \theta) = 2$$

for $-90^\circ < \theta < 90^\circ$. [3]

(iii) It is given that there are two values of θ , where $-90^\circ < \theta < 90^\circ$, satisfying the equation

$$6 \cos^2\left(\frac{1}{3}\theta + 45^\circ\right) - 3\left(\cos \frac{2}{3}\theta - \sin \frac{2}{3}\theta\right) = k,$$

where k is a constant. Find the set of possible values of k . [3]

Q5, (Jun 2015, Q9)

It is given that $f(\theta) = \sin(\theta + 30^\circ) + \cos(\theta + 60^\circ)$.

(i) Show that $f(\theta) = \cos \theta$. Hence show that

$$f(4\theta) + 4f(2\theta) \equiv 8 \cos^4 \theta - 3. \quad [6]$$

(ii) Hence

(a) determine the greatest and least values of $\frac{1}{f(4\theta) + 4f(2\theta) + 7}$ as θ varies, [3]

(b) solve the equation

$$\sin(12\alpha + 30^\circ) + \cos(12\alpha + 60^\circ) + 4 \sin(6\alpha + 30^\circ) + 4 \cos(6\alpha + 60^\circ) = 1$$

for $0^\circ < \alpha < 60^\circ$. [4]

Q6, (Jan 2006, Q9)

(i) By first writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta. \quad [4]$$

(ii) Determine the greatest possible value of

$$9 \sin\left(\frac{10}{3}\alpha\right) - 12 \sin^3\left(\frac{10}{3}\alpha\right),$$

and find the smallest positive value of α (in degrees) for which that greatest value occurs. [3]

(iii) Solve, for $0^\circ < \beta < 90^\circ$, the equation $3 \sin 6\beta \operatorname{cosec} 2\beta = 4$. [6]

Q7, (Jun 2016, Q9)

(i) Show that $\sin 2\theta(\tan \theta + \cot \theta) \equiv 2$. [4]

(ii) Hence

(a) find the exact value of $\tan \frac{1}{12}\pi + \tan \frac{1}{8}\pi + \cot \frac{1}{12}\pi + \cot \frac{1}{8}\pi$, [3]

(b) solve the equation $\sin 4\theta(\tan \theta + \cot \theta) = 1$ for $0 < \theta < \frac{1}{2}\pi$, [3]

(c) express $(1 - \cos 2\theta)^2 \left(\tan \frac{1}{2}\theta + \cot \frac{1}{2}\theta\right)^3$ in terms of $\sin \theta$. [2]

Q8, (Jun 2017, Q8)

(i) Express

$$3 \sin 2\theta \sec \theta + 4 \sin 2\theta \operatorname{cosec} \theta$$

in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [5]

(ii) Hence solve the equation

$$3 \sin(2\beta + 20^\circ) \sec(\beta + 10^\circ) + 4 \sin(2\beta + 20^\circ) \operatorname{cosec}(\beta + 10^\circ) = 3$$

for $0^\circ < \beta < 360^\circ$. [5]