Areas Involving Two Curves

Q1, (OCR 4722, Jan 2010, Q5)

$\int_1^3 \{(11-9x^{-2})-(x^2+1)\}dx = [9x^{-1} - \frac{1}{3}x^3 + 10x]_1^3$	M1		Attempt subtraction (correct order) at any point
$= (3 - 9 + 30) - (9 - {}^1/_3 + 10)$	M1		Attempt integration – inc. in power for at least one term
$= 24 - 18{}^2/_3$	A1		Obtain $\pm(-{}^1/_3x^3 + 10x)$ or $11x$ and ${}^1/_3x^3 + x$
$= 5{}^1/_3$	M1		Obtain remaining term of form kx^{-1}
OR	A1		Obtain $\pm 9x^{-1}$ or any unsimplified equiv
$[11x+9x^{-1}]_1^3 - [\frac{1}{3}x^3 + x]_1^3$	M1		Use limits $x = 1, 3$ – correct order & subtraction
$= [(33 + 3) - (11 + 9)] - [(9 + 3) - ({}^1/_3 + 1)]$ $= 16 - 10{}^2/_3$ $= 5{}^1/_3$	A1	7	Obtain $5{}^1/_3$, or exact equiv

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Q2, (OCR 4722, Jan 2012, Q7)

(a)		$\int\left(x^3-6x^2+4x-24\right)\mathrm{d}x$ $=\frac{1}{4}x^4-2x^3+2x^2-24x+c$	**M1** **A1ft** **A1** **[3]**	Expand and attempt in Obtain at least two correct (algebraic) terms Obtain fully correct expression, inc $+c$	Must attempt to expand brackets first Increase in power by 1 for the majority of their terms Allow if the constant term disappears At least two correct from their expansion Allow for unsimplified coefficients All coefficients now simplified A0 if integral sign or dx still present in their answer (but allow $\int = \ldots$)
(b)		$\int 6x^{\frac{3}{2}}\mathrm{d}x=\frac{12}{5}x^{\frac{5}{2}}$ $\int\left(8x^{-2}-2\right)\mathrm{d}x=-8x^{-1}-2x$ $\left[\frac{12}{5}x^{\frac{5}{2}}\right]_0^1=\frac{12}{5}$ $\left[-8x^{-1}-2x\right]_1^2=(-8)-(-10)=2$ hence total area $=\frac{22}{5}$	**M1** **A1** **M1** **A1** **B1** **M1**	Obtain $kx^{\frac{5}{2}}$ Obtain $\frac{12}{5}x^{\frac{5}{2}}$, or any exact equiv Obtain at least one of $-8x^{-1}$ and $-2x$ Obtain $-8x^{-1}-2x$ State or imply that pt of intersection is (2, 0) Use limits correctly at least once	Any exact equiv for the index Including unsimplified coefficient Allow M1 even if -2 disappears Could be part of a sum or difference; with consistent signs Allow unsimplified expressions If subtraction from other curve attempted before integration then allow for $8x^{-1}+2x$ Could imply by using it as a limit Must be using correct x limits, and subtracting, with the appropriate function (allow implicit use of $x = 0$); the only error allowed is an incorrect (2, 0) Allow use in any function other than the original, inc from differentiation

Q3, (OCR 4723, Jun 2017, Q5)

Differentiate to obtain $k(4x-3)^{-\frac{1}{2}}$	M1	For any non-zero constant k
Obtain correct $2(4x-3)^{-\frac{1}{2}}$	A1	Or unsimplified equiv
Use negative reciprocal of gradient to find intersection of normal with x-axis	M1	Using their attempt at first derivative; either using equation of normal $(y=-\frac{5}{2}x+\frac{45}{2})$ or relevant right-angled triangle
Obtain $-\frac{5}{2}$ for gradient of normal and hence $x=9$ or equiv such as base of triangle is 2	A1	
Integrate to obtain $p(4x-3)^{\frac{3}{2}}$	M1	For any non-zero constant p
Obtain correct $\frac{1}{6}(4x-3)^{\frac{3}{2}}$	A1	Or unsimplified equiv
Use limits $\frac{3}{4}$ and 7 to obtain $\frac{125}{6}$ for area under curve	A1	Allow calculation apparently using only upper limit
Use triangle area to obtain $\frac{155}{6}$ for shaded area	A1	
	[8]	

Q4, (OCR 4722, Jun 2017, Q6)

$\int(11-x-2x^2)dx = 11x-\frac{1}{2}x^2-\frac{2}{3}x^3$ $\int 8x^{-3}dx = -4x^{-2}$	M1	Attempt integration of $11-x-2x^2$	Increase in power by 1 for at least 2 terms
	A1	Obtain $11x-{}^1/_2x^2-{}^2/_3x^3$	Obtain correct integral
$(22-2-{}^{16}/_3)-(11-{}^1/_2-{}^2/_3)={}^{29}/_6$	M1	Attempt integration of $8x^{-3}$	Integrate to kx^{-2}
$(-1)-(-4)=3$	A1	Obtain $-4x^{-2}$	Allow unsimplified coeff
${}^{29}/_6-3={}^{11}/_6$	M1	Use limits of $x=1, 2$	In both integrals Must follow clear attempt at integration Must be F(2) – F(1) ie correct order and subtraction
	M1	Attempt correct method to find shaded area (at any point)	M0 if incorrect order of subtraction, even if ${}^{11}/_6$ subsequently appears as final answer M1 can follow M0 for use of limits
	A1	Obtain ${}^{11}/_6$, or exact equiv	A0 for decimal answer unless clearly a recurring decimal (but not eg 1.833...) ISW if ${}^{11}/_6$ seen but then followed by eg 1.83
	[7]		Answer only is 0/7 - need to see evidence of integration, but use of limits does not need to be explicit

Q5, (OCR 4723, Jun 2016, Q5)

i		Either State $e^{2x} = 8e^{-x}$ and so $e^{3x} = 8$	B1		Verifying by substitution of $\ln 2$ in each equation earns B0B0
		Obtain $e^{x} = 2$ and hence $x = \ln 2$	B1	AG; necessary detail needed	
		Or 1 State $e^{2x} = 8e^{-x}$ and so $e^{3x} = 8$	B1		
		State $3x = \ln 8$, $x = \ln 8^{\frac{1}{3}}$ and hence $x = \ln 2$	B1	AG; necessary detail needed	Going immediately from $x = \frac{1}{3}\ln 8$ to $x = \ln 2$ does not earn the second B1
		Or 2 State $e^{2x} = 8e^{-x}$ and $2x = \ln 8 - x$	B1		
		State $3x = \ln 8$, $x = \ln 8^{\frac{1}{3}}$ and hence $x = \ln 2$	B1	AG; necessary detail needed	Going immediately from $x = \frac{1}{3}\ln 8$ to $x = \ln 2$ does not earn the second B1
			[2]		
ii		Integrate to obtain k_1e^{-x} and k_2e^{2x}	M1	Any non-zero constants k_1 and k_2	
		Obtain correct $-8e^{-x} - \frac{1}{2}e^{2x}$ or, if done separately, $-8e^{-x}$ and $\frac{1}{2}e^{2x}$	A1		
		Apply limits 0 and $\ln 2$ correctly to their integral(s)	M1	Condone one sign slip; earned by sight of $-8e^{-\ln 2} - \frac{1}{2}e^{2\ln 2} + 8 + \frac{1}{2}$ (or equivs if integrals treated separately)	M1 also implied by sight only of $-4 - 2 + 8 + \frac{1}{2}$ (or equivs …)
		Obtain at least $-4 - 2 + 8 + \frac{1}{2}$ (or equivs)	*A1		
		Obtain $\frac{5}{2}$ or equiv	A1	Final A1 dependent on *A1	
			[5]		

Q6, (OCR 4723, Jun 2005, Q8)

(i)	Attempt relevant calculations with 5.2 and 5.3	**M1**	
	Obtain correct values	**A1**	x y_1 y_2 $y_1 - y_2$ 5.2 2.83 2.87 – 0.04 5.3 2.89 2.88 0.006
	Conclude appropriately	**A1**	**3** [**AG**; comparing y values or noting sign change in difference in y values or equiv]
(ii)	Equate expressions and attempt rearrangement to $x =$	**M1**	
	Obtain $x = \frac{5}{3}\ln(3x+8)$	**A1**	**2** [**AG**; necessary detail required]
(iii)	Obtain correct first iterate	**B1**	
	Carry out correct process to find at least two iterates in all	**M1**	
	Obtain 5.29	**A1**	**3** [must be exactly 2 decimal places; 5.2→5.2687→5.2832→5.2863→5.2869; 5.25→5.2793→5.2855→5.2868→5.2870; 5.3→5.2898→5.2877→5.2872→5.2871]
(iv)	Obtain integral of form $k(3x+8)^{\frac{4}{3}}$	**M1**	
	Obtain integral of form $k\,e^{\frac{1}{5}x}$	**M1**	