

**Binomial Hypothesis Testing Problems Longer Exam Questions MS**

**Q1 (Jun 2016, Q7)**

(i)	(A)	$X \sim B(16, 0.1)$ $P(X = 3) = 0.25^4 \times 0.75^{16} \binom{20}{4} \times 0.25^4 \times 0.75^{16} = 0.1423$  Or: From tables $P(X \leq 3) - P(X \leq 2) = 0.9316 - 0.7892 = 0.1424$	M1 M1 A1 M2 A1 <b>[3]</b>	For $0.1^3 \times 0.9^{13}$ For $\binom{16}{3} \times p^3 \times q^{13}$ CAO For $0.9316 - 0.7892$ CAO	With $p + q = 1$ Also for $560 \times 0.000254..$ Allow 0.14 or better
(i)	(B)	$P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0.7892 = 0.2108$	M1 A1 <b>[2]</b>	For 0.7892 CAO	If calculating $P(X = 0) + P(X = 1) + P(X = 2)$ allow M1 for 0.79 or better and A1 for 0.21 or better.

(i)	(C)	Expected number = $16 \times 0.1 = 1.6$	B1 [1]		Do not allow final answer of 1 or 2 even if correct 1.6 given earlier
(ii)		Let $p$ = probability of a randomly chosen person using 1234 as their PIN (in the population) $H_0: p = 0.1$ $H_1: p < 0.1$  The alternative hypothesis has this form as the advertising campaign aims to reduce the proportion of the population who use 1234 as their PIN.	B1  B1 B1  B1 [4]	For definition of $p$ (in context)  For $H_0$ For $H_1$  Dep on $< 0.1$ used in $H_1$ Do Not allow just 'proportion will be lower' or similar.	Do NOT allow <u>number</u> in place of probability. See below for additional notes  For use of 0.9 as P(do not use 1234), contact team leader. E0 for simply stating $H_1$ in words Condone number instead of proportion.
(iii)	(A)	For $n = 20$ , $P(X \leq 0) = 0.1216$  $0.1216 > 0.10$  So no point in carrying out the test as $H_0$ could not be rejected (even if nobody in the sample uses 1234 as their PIN). oe	M1*  *M1dep  A1 [3]	For sight of 0.1216  For $> 0.10$ or $> 10\%$ Do NOT FT wrong $H_1$  or state 'There is no critical region' oe For A1 need $P(X \leq 0)$ or $P(X = 0)$ somewhere oe	Condone $P(X = 0)$ in place of $P(X \leq 0)$ Need to see a comparison with 0.1 or 10% explicitly, not just mentioning significance level. Allow SC2 for clearly indicating use of $B(20, 0.1)$ but with no mention of 0.1216 with convincing reasoning and final answer correct Allow CR is empty but NOT CR is zero
	(B)	Lowest value of $k$ is 13	B1 [1]	Or 13%	
(iv)		$P(X \leq 2) = 0.0530$ $0.0530 > 0.05$  So not significant. Do not reject $H_0$ Conclude that there is not enough evidence to support the suggestion that the advertising campaign has been successful.  Reminder: When you mark this question part, if you 'fit to height' you can check the last page for working or mark it BP if there none	B1 M1  A1* *E1 dep [4]	For <u>use</u> of $P(X \leq 2)$ only For comparison of 0.0530 with 5%  Also allow $P(X \leq 2) > 0.05$ , $(P(X \leq 1) < 0.05)$ so CR is $\{0, 1\}$ for first two marks then A1E1 as usual Condone 'number of people' in conclusion	No marks unless $H_1$ correct If B0 then no further marks  Allow 'accept $H_0$ ' or 'reject $H_1$ ' Must include 'insufficient evidence to suggest that' or something similar i.e. an element of doubt either in the A or E mark.

**Q2 (Jun 2015, Q7)**

(i)	(B)	$P(\text{Exactly 20 cured}) = \binom{20}{20} \times 0.78^{20} \times 0.22^0 = 0.0069$ $P(\text{At most 18 cured}) = 1 - (0.0069 + 0.0392)$ $= 0.954 \text{ (0.95385)}$	M1          A1 <b>[3]</b>	For $0.78^{20}$ oe          CAO	Allow M2 for 0.9488 for linear interpolation from tables or M1 for $1 - 0.9918 = 0.0082$ and second M1 for correct FT using answer to (i)(A) Zero for use of $p = 0.8$ here Not necessarily correct, but both attempts at binomial, including coefficient in (i) and no extra terms (such as $P(X=18)$ ) Condone use of $p = 0.8$ Allow 0.95 with working
(i)	(C)	$E(X) = np = 20 \times 0.78 = 15.6$	B1          <b>[1]</b>	CAO	Do not allow final answer of 15 or 16 even if correct 15.6 given earlier
(ii)		<p>Let <math>X \sim B(20, 0.78)</math></p> <p>Let <math>p =</math> probability of a patient being cured (for population)</p> <p><math>H_0: p = 0.78</math> <math>H_1: p &gt; 0.78</math></p>	B1          B1 B1	For definition of $p$          For $H_0$ For $H_1$	In context See below for additional notes          No further marks if point probabilities

	<p><math>P(X \geq 19) = 0.0392 + 0.0069</math></p> <p><math>= 0.0461</math></p> <p><math>0.0461 &gt; 1\%</math></p> <p>So not significant. Conclude that there is not enough evidence to suggest that the new drug is more effective than the old one.</p>	<p>B1</p> <p>B1*</p> <p>MI* dep A1 E1</p> <p>[8]</p>	<p>For <b>NOTATION</b> <math>P(X \geq 19)</math> or <math>P(X &gt; 18)</math> or <math>1 - P(X \leq 18)</math> or <math>1 - P(X &lt; 19)</math></p> <p>CAO For 0.0461 allow 0.0462</p> <p>For comparison with 1%</p>	<p>used Notation <math>P(X = 19)</math> scores B0. If they have the correct <math>P(X \geq 19)</math> then give B1 and ignore any further incorrect notation.</p> <p>FT answer to (i)B for following three marks provided based on <math>1 - (P(19) + P(20))</math> Dep on sensible attempt at <math>P(X \geq 19)</math></p> <p>Allow 'accept <math>H_0</math>' or 'reject <math>H_1</math>' Must include 'insufficient evidence' or something similar such as 'to suggest that' ie an element of doubt either in the A or E mark. Must be in context to gain E1 mark. Do NOT allow 'sufficient evidence to suggest proportion cured is 0.78' or similar <u>99% method:</u> <math>P(X \leq 18) = 0.9539</math> B1B1* CAO <math>0.9539 &lt; 99\%</math> MI* then as per scheme</p>
(iii)	<p>is more effective than the old one. With a 5% significance level rather than a 1% level, the null hypothesis would have been rejected. OR: 'there would be enough evidence to suggest that the new drug is more effective than the old one.' This is because <math>0.0461 &lt; 5\%</math></p>	<p>B1*</p> <p>B1* dep</p> <p>[2]</p>	<p>oe</p> <p>oe</p>	<p>FT their probability from (ii) but NO marks if point probabilities used There must be a sensible attempt to use <math>P(X = 19) + P(X = 20)</math> or must have correct CR. Dep on correct answer of 0.0461 compared with 5% or 0.9539 compared with 95% or correct CR.</p>

**Q3 (Jun 2014, Q7)**

(i)	(A)	$X \sim B(15, 0.85)$	M1	For $0.85^{12} \times 0.15^3$	
		$P(\text{exactly 12 germinate}) = \binom{15}{12} \times 0.85^{12} \times 0.15^3$	M1	For $\binom{15}{12} \times p^{12} \times q^3$	
		= 0.2184	A1	CAO	
		<b>OR</b>	<b>OR</b>		
		from tables: $0.3958 - 0.1773$	M2	For $0.3958 - 0.1773$	
		= 0.2185	A1	CAO	
			<b>[3]</b>		
(i)	(B)	$P(X < 12) = P(X \leq 11) = 0.1773$	M1	For $P(X \leq 11)$ or $P(\leq 11)$ (With no extras) CAO (as final answer)	
			A1	May see alternative method: $0.3958 - 0.2185 = 0.1773$ $0.3958$ - their wrong answer to part (i) scores M1A0	
			<b>[2]</b>		

<b>(ii)</b>	<p>Let <math>p</math> = probability of a seed germinating (for the population)</p> <p><math>H_0: p = 0.85</math>  <math>H_1: p &lt; 0.85</math></p> <p><math>H_1</math> has this form because the test is to investigate whether the proportion of seeds which germinate is lower.</p>	<p>B1  B1  B1  E1  <b>[4]</b></p>	<p>For definition of <math>p</math></p> <p>For <math>H_0</math></p> <p>For <math>H_1</math></p> <p>Dep on <math>&lt; 0.85</math> used in <math>H_1</math>  Do not allow just 'Germination rate will be lower' or similar.</p>	<p>See below for additional notes</p> <p>For use of 0.15 as P(not germinating), contact team leader  E0 for simply stating <math>H_1</math> in words</p>
<b>(iii)</b>	<p>Let <math>X \sim B(20, 0.85)</math>  <math>P(X \leq 13) = 0.0219</math></p> <p><math>0.0219 &gt; 1\%</math></p> <p>So not enough evidence to reject <math>H_0</math>.  Not significant.</p> <p>Conclude that there is not enough evidence to indicate that the proportion of seeds which have germinated has decreased.</p>	<p>M1*  M1*  dep  A1*  E1*  dep</p>	<p>For probability (provided not as part of finding <math>P(X = 13)</math>) Ignore notation</p> <p>For comparison</p> <p>For not significant oe</p> <p>For conclusion in context  Must mention decrease, not just change</p>	<p>No further marks if point probs used - <math>P(X = 13) = 0.0160</math>  DO NOT FT wrong <math>H_1</math>, but see extra notes  Allow 'accept <math>H_0</math>' or 'reject <math>H_1</math>'  Must include 'sufficient evidence' or something similar such as 'to suggest that' ie an element of doubt either in the A or E mark.</p>

(iv)	<p><math>33 &lt; 35</math></p> <p>So there is sufficient evidence to reject <math>H_0</math></p> <p>Conclude that there is enough evidence to indicate that the proportion of seeds which have germinated has decreased.</p>	<p>M1</p> <p>A1*</p> <p>E1* dep</p> <p>[3]</p>	<p>For comparison</p> <p>For conclusion in context</p> <p>Must mention decrease, not just change</p>	<p>Allow '33 lies in the CR' Must include 'sufficient evidence' or something similar such as 'to suggest that' ie an element of doubt either in the A or E mark.</p> <p>Do not FT wrong <math>H_1</math>: In part (iv) ignore any interchanged <math>H_0</math> and <math>H_1</math> seen in part (ii)</p> <p>If use a calculator to find <math>P(X \leq 33) = 0.000661</math> and compare with 1% then B2 for <math>P(X \leq 33) = 0.000661 &lt; 0.01</math> so reject <math>H_0</math> then final E1 as per scheme.</p>
(v)	<p>For <math>n = 3</math>, <math>P(X \leq 0) = 0.0034 &lt; 0.01</math> For <math>n = 2</math>, <math>P(X \leq 0) = 0.0225 &gt; 0.01</math></p> <p>So the least value of <math>n</math> for which the critical region is not empty and thus <math>H_0</math> could be rejected is 3.</p> <p>ALTERNATIVE METHOD using logs <math>0.15^n &lt; 0.01</math> <math>n &gt; \log 0.01 / \log 0.15</math> <math>n &gt; 2.427</math> Least <math>n = 3</math></p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>For <math>P(X \leq 0) = 0.0034</math> For <math>P(X \leq 0) = 0.0225</math></p> <p>CAO</p>	<p>Allow 0.003</p> <p>Condone just '<math>n = 3</math>' for final A mark dep on both M marks</p> <p>If wrong <math>H_1</math> allow max M2A0 if correct probabilities seen.</p>

**Q4 (Jan 2013, Q7)**

(i)	(A)	$X \sim B(10, 0.35)$ $P(5 \text{ accessing internet}) = \binom{10}{5} \times 0.35^5 \times 0.65^5$  $= 0.1536$  <b>OR</b> from tables $= 0.9051 - 0.7515 = 0.1536$	M1  M1  A1   <b>OR</b> M2 A1 <b>[3]</b>	or $0.35^5 \times 0.65^5$ For $\binom{10}{5} \times p^5 \times q^5$  cao   For $0.9051 - 0.7515$ cao	With $p + q = 1$ Also for $252 \times 0.0006094$  Allow 0.15 or better NB 0.153 gets A0 See tables at the website <a href="http://www.mei.org.uk/files/pdf/formula_book_mf2.pdf">http://www.mei.org.uk/files/pdf/formula_book_mf2.pdf</a>
(i)	(B)	$P(X \geq 5) = 1 - P(X \leq 4)$ $= 1 - 0.7515$ $= 0.2485$	M1 A1   <b>[2]</b>	For 0.7515 cao	Accept 0.25 or better – allow 0.248 or 0.249 Calculation of individual probabilities gets B2 if fully correct 0.25 or better, otherwise B0.
(i)	(C)	$E(X) = np = 10 \times 0.35$  $= 3.5$	M1  A1 <b>[2]</b>	For $10 \times 0.35$  cao	If any indication of rounding to 3 or 4 allow M1A0



(ii)	<p>Let <math>X \sim B(20, 0.35)</math>                  Let <math>p</math> = probability of a customer using the internet (for population)</p>	B1	For definition of $p$ in context	Minimum needed for B1 is $p$ = probability of using internet. Allow $p = P(\text{using internet})$ Definition of $p$ must include word probability (or chance or proportion or percentage or likelihood but NOT possibility). Preferably as a separate comment. However can be at end of $H_0$ as long as it is a clear definition 'p = the probability of using internet', Do NOT allow 'p = the probability of using internet is different'
	<p><math>H_0: p = 0.35</math></p>	B1	For $H_0$	Allow $p=35\%$ , allow only $p$ or $\theta$ or $\pi$ or $\rho$ . However allow any single symbol <u>if defined</u> (including $x$ ) Allow $H_0 = p=0.35$ , Allow $H_0: p=7/20$ or $p=35/100$ Allow NH and AH in place of $H_0$ and $H_1$ Do not allow $H_0: P(X=x) = 0.35$ Do not allow $H_0: =0.35, =35\%, P(0.35), p(x)=0.35, x=0.35$ (unless $x$ correctly defined as a probability) Do not allow $H_0$ and $H_1$ reversed For hypotheses given in words allow Maximum B0B1B1 Hypotheses in words must include probability (or chance or proportion or percentage) and the figure 0.35 or Thus eg $H_0: p(\text{using internet}) = 0.35$ , $H_1: p(\text{using internet}) \neq 0.35$ gets B0B1B1

<p><math>H_1: p \neq 0.35</math></p> <p><math>H_1</math> has this form because the test is to investigate whether the proportion is different, (rather than lower or higher).  <math>P(X \geq 10)</math></p> <p><math>= 1 - 0.8782 = 0.1218</math></p> <p><math>&gt; 2.5\%</math></p> <p>So not significant.          Conclude that there is not enough evidence to indicate that the probability is different. (Must state 'probability', not just 'p')</p>	<p>B1</p> <p>E1</p> <p>B1</p> <p>B1*</p> <p>M1*</p> <p>dep</p> <p>A1*</p> <p>E1*</p> <p>dep on</p> <p>A1</p>	<p>For <math>H_1</math></p> <p>For notation <math>P(X \geq 10)</math> or <math>P(X &gt; 9)</math> or <math>1 - P(X \leq 9)</math> (as long as no incorrect notation)</p> <p>For 0.1218 Allow 0.12</p> <p>For comparison with 2.5%</p>	<p>Allow '<math>p &lt; 0.35</math> or <math>p &gt; 0.35</math>' in place of <math>p \neq 0.35</math>          Do not allow if <math>H_1</math> wrong.</p> <p>This mark may be implied by 0.1218 as long as no incorrect notation.          No further marks if point probs used - <math>P(X = 10) = 0.0686</math> (do not even give the notation mark for correct notation)          DO NOT FT wrong <math>H_1</math>, but see extra notes          Or for <math>1 - 0.8782</math>          Indep of previous mark</p> <p>Allow 'accept <math>H_0</math>' or 'reject <math>H_1</math>'          Must include 'sufficient evidence' or something similar such as 'to suggest that' ie an element of doubt either in the A or E mark.</p>	
<p>(iii)</p>	<p><math>0.0022 &lt; 2.5\%</math>          So reject <math>H_0</math>,          Significant.</p> <p>Conclude that there is enough evidence to indicate that the probability is different.</p>	<p>B1</p> <p>E1*</p> <p>dep</p> <p>[2]</p>	<p>For either reject <math>H_0</math> or significant, dep on correct comparison</p> <p>Dep on good attempt at correct hypotheses in part (ii)</p>	<p>If they have <math>H_1: p &gt; 0.35</math>, allow SC1 if all correct including comparison with 5%.</p>

**Q5 (Jun 2013, Q5)**

<b>(i)</b>	<p>Because if people cannot make a correct identification, then the probability that they guess correctly will be 0.5                  For 'equally likely to guess right or wrong' or 'two outcomes with equal probability' or '50:50 chance of success' or 'right one in two occasions on average' or 'two (equally likely) outcomes' etc</p>	<p>E1                  E1                    [2]</p>	<p>For idea of a guess or 'chosen at random'                  For idea of two outcomes</p>	<p>NB The question includes the sentence 'She suspects that people do no better than they would by guessing.', so this on its own does not get the mark for the idea of a guess</p>
<b>(ii)</b>	<p>'Because people may do better than they would by guessing' or similar</p>	<p>B1                    [1]</p>	<p>For idea of selecting correctly /identifying /knowing</p>	<p>No marks if answer implies that it is because there are over half in the sample who make a correct identification</p>
<b>(iii)</b>	<p><math>P(X \geq 13) = 1 - P(X \leq 12) = 1 - 0.8684 = 0.1316</math>    <b>NB PLEASE ANNOTATE THE TOP AND BOTTOM OF THE EXTRA PAGE IF NOT USED</b>    <math>0.1316 &gt; 0.05</math>                    So not significant                    There is insufficient evidence to suggest that people can make a correct identification.</p>	<p>M1                    B1*                    M1*                  dep                  A1*                    E1*                  dep</p>	<p>For notation <math>P(X \geq 13)</math> or <math>P(X &gt; 12)</math> or <math>1 - P(X \leq 12)</math>                    For 0.1316                    For comparison with 5%</p> <p>NB Point probabilities score zero.</p>	<p>Notation <math>P(X = 13)</math> scores M0.                  If they have the correct <math>P(X \geq 13)</math> then give M1 and ignore any further incorrect notation.                  Or for <math>1 - 0.8684</math> indep of previous mark                    Allow 'accept <math>H_0</math>' or 'reject <math>H_1</math>'                    Must include 'insufficient evidence' or something similar such as 'to suggest that' ie an element of doubt either in the A or E mark. Must be in context to gain E1 mark.                  Do not allow 'sufficient evidence to suggest proportion making correct identification is 0.5' or similar</p>

Q6 (Jun 2012, Q5)

<p>Let <math>p</math> = probability that a randomly selected frame is faulty</p> <p><math>H_0: p = 0.05</math></p> <p><math>H_1: p &gt; 0.05</math>  <math>P(X \geq 4)</math></p> <p>= <math>1 - P(X \leq 3) = 1 - 0.9891 = 0.0109</math></p>	<p>B1</p> <p>For definition of <math>p</math> in context          Minimum needed for B1 is <math>p</math> = probability that frame/bike is faulty. Do not allow is <math>p</math> = probability that it is faulty          Allow <math>p = P(\text{frame faulty})</math>          Definition of <math>p</math> must include word probability (or chance or proportion or percentage or likelihood but NOT possibility). Preferably as a separate comment. However can be at end of <math>H_0</math> as long as it is a clear definition ' <math>p</math> = the probability that frame is faulty, NOT just a sentence 'probability is 0.05'          Do NOT allow ' <math>p</math> = the probability that faulty frames have increased'</p> <p>B1</p> <p><math>H_0: p(\text{frame faulty}) = 0.05, H_1: p(\text{frame faulty}) &gt; 0.05</math> gets B0B1B1          Allow <math>p=5\%</math>, allow <math>\theta</math> or <math>\pi</math> and <math>\rho</math> but not <math>x</math>. However allow any single symbol if <u>defined</u>          Allow <math>H_0 = p=0.05</math>, Allow <math>H_0: p=1/20</math>          Do not allow <math>H_0: P(X=x) = 0.05, H_1: P(X=x) &gt; 0.05</math>          Do not allow <math>H_0: =0.05, =5\%, P(0.05), p(0052), p(x)=0.05, x=0.05</math> (unless <math>x</math> correctly defined as a probability)          Do not allow <math>H_1: p \geq 0.05</math>,          Do not allow <math>H_0</math> and <math>H_1</math> reversed          Allow NH and AH in place of <math>H_0</math> and <math>H_1</math>          For hypotheses given in words allow Maximum B0B1B1          Hypotheses in words must include probability (or chance or proportion or percentage) and the figure 0.05 oe.</p>	<p>B1</p> <p>B1</p> <p>For notation <math>P(X \geq 4)</math> or <math>1 - P(X \leq 3)</math>          This mark may be implied by 0.0109 as long as no incorrect notation.</p> <p>B1*</p> <p>For 0.0109, indep of previous mark</p> <p>No further marks if point probs used - <math>P(X = 4) = 0.0094</math>          DO NOT FT wrong <math>H_1</math>          But if <math>H_1</math> is <math>p \geq 0.05</math> allow the rest of the marks if earned so max 7/8          Or for <math>1 - 0.9891</math></p>
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<p><math>0.0109 &lt; 0.05</math></p> <p>So reject <math>H_0</math></p> <p>There is evidence to suggest that the proportion of faulty frames has increased.</p> <p><b>OR Critical region method:</b>          Let <math>X \sim B(18, 0.05)</math>  <math>P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0.9419 = 0.0581 &gt; 5\%</math>  <math>P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.9891 = 0.0109 &lt; 5\%</math></p> <p>So critical region is <math>\{4,5,6,7,8,9,10,11,12,13,14,15,16,17,18\}</math>          4 lies in the critical region, so significant,</p> <p>There is evidence to suggest that the proportion of faulty frames has increased.</p>	<p>M1* dep A1*</p> <p>E1* Dep on A1</p> <p><b>[8]</b></p> <p>(B1)</p> <p>(B1)</p> <p>(M1)</p> <p>(A1)</p> <p>(E1)</p>	<p>For comparison with 5% or significant or 'accept <math>H_1</math>'</p> <p>Must include 'sufficient evidence' or something similar such as 'to suggest that' ie an element of doubt for E1. 'Sufficient evidence' or similar can be seen in the either the A mark or the E mark.</p> <p>For 0.0581</p> <p>For 0.0109</p> <p>For at least one correct comparison with 5%</p> <p>CAO for critical region and significant oe</p>	<p>No marks if CR not justified Do not insist on correct notation as candidates have to work out two probabilities for full marks</p> <p>Condone <math>\{4,5 \dots\}</math>, <math>X \geq 4</math>, oe but not <math>P(X \geq 4)</math></p>
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