

Binomial Hypothesis Testing (From OCR 4733)

Q1 (Jan 2006, Q3)

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| $H_0: p = 0.3$ | B1 | NH stated, must be this form (or π) |
| $H_1: p \neq 0.3$ | B1 | AH stated, must be this form (or π) [μ : B1 both] |
| $B(8, 0.3)$ | M1 | $B(8, 0.3)$ stated or implied |
| $P(\leq 4) = 0.9420;$ $P(> 4) = 0.0580$ | A1 M1 | Any one of these four probabilities seen |
| $P(\leq 5) = 0.9887;$ $P(> 5) = 0.0113$ | M1 | Either compare $P(\geq 5) \& 0.025 / P(\leq 4) \& 0.975$ |
| Compare 0.025 or critical value 6 | | Or critical region ≥ 6 with 5 |
| Do not reject H_0 | A1√ | H_0 not rejected, can be implied, needs essentially correct method |
| Insufficient evidence that manufacturer's claim is wrong | 7 | Correct conclusion in context [SR: Normal, Poisson: can get B2M1A0M0M1A1 $P(\leq 5)$: first 4 marks. $P(= 5)$: first 3 marks only.] |

Q2, (Jun 2006, Q2)

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| (i) Find $P(R \geq 6)$ or $P(R < 6)$ $= 0.0083$ or 0.9917 | M1 A1 | Find $P(= 6)$ from tables/calc, OR RH critical region |
| Compare with 0.025 [can be from N] | B1 | $P(\geq 6)$ in range [0.008, 0.0083] or $P(< 6) = 0.9917$ |
| [0.05 if "empty LH tail stated] | A1√ | OR CR is 6 with probability 0.0083/0.9917 |
| Reject H_0 | 4 | Explicitly compare with 0.025 [or 0.975 if consistent] OR state that result is in critical region |
| (ii) $n = 9, P(\leq 1) = 0.0385$ [> 0.025] $n = 10, P(\leq 1) = 0.0233$ [< 0.025] Therefore $n = 9$ | M1 A1 B1 | Correct comparison and conclusion, √ on their p At least one, or $n = 8, P(\leq 1) = 0.0632$ Both of these probabilities seen, don't need 0.025 |
| | 3 | Answer $n = 9$ only, indep't of M1A1, not from $P(= 1)$ |

Q3, (Jan 2007, Q7)

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| (i) $H_0: p = 0.35$ [or $p \geq 0.35$] $H_1: p < 0.35$ $B(14, 0.35)$ | B1 B1 M1 | Each hypothesis correct, B1+B1, allow $p \geq .35$ if .35 used [Wrong or no symbol, B1, but r or x or \bar{x} : B0] |
| $\alpha: P(\leq 2) = 0.0839 > 0.025$ | | Correct distribution stated or implied, can be implied by $N(4.9, \dots)$, but not $Po(4.9)$ |
| $\beta: CR \leq 1$, probability 0.0205 | A1 | 0.0839 seen, or $P(\leq 1) = 0.0205$ if clearly using CR |
| Do not reject H_0 . Insufficient evidence that proportion that can receive Channel C is less than 35% | B1 M1 A1√ | Compare binomial tail with 0.025, or $R = 2$ binomial CR Do not reject H_0 , √ on their probability, not from N or Po or $P(< 2)$; Contextualised conclusion √ |
| (ii) $B(8, 0.35): P(0) = 0.0319$ $B(9, 0.35): P(0) = 0.0207$ | M1 A1 A1 | Attempt to find $P(0)$ from $B(n, 0.35)$ One correct probability [$P(\leq 2) = .0236, n = 18$: M1A1] |
| Hence largest value of n is 8 | A1 | Both probabilities correct |
| or $0.65^n > 0.025; n \ln 0.65 > \ln 0.025$ $8.56;$ largest value of $n = 8$ | A1√ | Answer 8 or ≤ 8 only, needs minimum M1A1 |
| | M1M1 A1A1 | $p^n > 0.025$, any relevant p ; take ln, or T&I to get 1 SF In range [8.5, 8.6]; answer 8 or ≤ 8 only |

Q4, (Jun 2007, Q6)

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| (i) | $H_0: p = 0.19, H_1: p < 0.19$ where p is population proportion $0.81^{20} + 20 \times 0.81^{19} \times 0.19$ $= 0.0841$ Compare 0.1 | B2 M1 A1 A1 B1 | | Correct, B2. One error, B1, but x or \bar{x} or r : B0 Binomial probabilities, allow 1 term only Correct expression [0.0148 + 0.0693] Probability, a.r.t. 0.084 Explicit comparison of "like with like" |
| or | Add binomial probs until ans > 0.1 Critical region ≤ 1 | A1 B1 | | [$P(\leq 2) = 0.239$] |
| | Reject H_0 Significant evidence that proportion of e 's in language is less than 0.19 | M1 A1√ | 8 | Correct deduction and method [needs $P(\leq 1)$] Correct conclusion in context [SR: N(3.8, 3.078): B2M1A0B1M0] |
| (ii) | Letters not independent | B1 | 1 | Correct modelling assumption, stated in context Allow "random", "depends on message", etc |

Q5, (Jan 2008, Q8)

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| (i) | $H_0: p = 0.65$ OR $p \geq 0.65$ $H_1: p < 0.65$ B(12, 0.65) | B2 M1 | | Both hypotheses correctly stated, in this form [One error (but not r, x or \bar{x}): B1] B(12, 0.65) stated or implied |
| | α : $P(\leq 6) = 0.2127$ Compare 0.10 | A1 B1 | | Correct probability from tables, <i>not</i> $P(= 6)$ Explicit comparison with 0.10 |
| | β : Critical region $\leq 5; 6 > 5$ Probability 0.0846 | B1 A1 | | Critical region ≤ 5 or ≤ 6 or $\{\leq 4\} \cap \{\geq 11\}$ & compare 6 Correct probability |
| | Do not reject H_0 Insufficient evidence that proportion of population in favour is not at least 65% | M1√ A1√ | 7 | Correct comparison and conclusion, needs correct distribution, correct tail, like-with-like Interpret in context, e.g. "consistent with claim" [SR: N(7.8, 2.73): can get B2M1A0B1M0: 4 ex 7] |
| (ii) | Insufficient evidence to reject claim; test and p/q symmetric | B1√ B1 | 2 | Same conclusion as for part (i), don't need context Valid relevant reason, e.g. "same as (i)" |
| (iii) | $R \sim B(2n, 0.65), P(R \leq n) > 0.15$ B(18, 0.65), $p = 0.1391$ Therefore $n = 9$ | M1 A1 A1 A1 | 4 | B($2n, 0.65$), $P(R \leq n) > 0.15$ stated or implied Any probability in list below seen $p = 0.1391$ picked out (i.e., not just in a list of > 2) Final answer $n = 9$ only [SR $< n$: M1A0, $n = 4, 0.1061$ A1A0] [SR 2-tail: M1A1A0A1 for 15 or 14] [SR: 9 only, no working: M1A1] [MR B(12, 0.35): M1A0, $n = 4, 0.1061$ A1A0] 3 0.3529 7 0.1836 12 0.0942 4 0.2936 8 0.1594 13 0.0832 5 0.2485 9 0.1391 14 0.0736 6 0.2127 10 0.1218 15 0.0652 |

Q6, (Jan 2011, Q9)

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| (i) | B(14, 0.7) CR is ≥ 13 with probability 0.0475 | M1 A1 A1 | 3 | B(14, 0.7) stated or implied, e.g. N(9.8, 2.94), can be recovered CV 13, or > 12 or {13, 14}, allow = but no other inequalities Exactly correct CR, and supporting prob .0475 or .9525 seen |
| (ii) | $H_0: p = 0.7, H_1: p > 0.7$ $12 < 13$ Do not reject H_0 . Insufficient evidence that proportion who show improvement is greater than 0.7 | B2 B1 M1 A1 FT | 5 | Both, B2. Allow π . One error, B1, but r, x etc: B0 Compare CV from correct tail and inequality with 12, or $P(\geq 12) = 0.1608$ and > 0.05 or $P(< 12) = 0.8392$ and < 0.95 Correct method & conclusion, requires like-with-like; CV method needs ≥ 13 or < 12 ; p method needs ≥ 12 or < 12 Withhold if inconsistent Contextualised, acknowledge uncertainty [SR: Normal or Po: (i) M1, (ii) B2 maximum] [0.9932 or 0.0068 probably B2 maximum] |
| (iii) | B(14, 0.8) $P(\leq 12)$ from B(14, 0.8) 0.8021 | M1 M1 A1 | 3 | B(14, 0.8) stated or implied, allow from B(14, 0.75) Attempt prob of acceptance region, e.g. 0.8990, $\sqrt{\quad}$ on (i) Answer 0.802 or a.r.t. 0.8021 |

Q7, (Jun 2014, Q6)

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| (i) | Sample is random | B1 [1] | Indicate random sample. Allow “unbiased sample” or “randomly selected” or “all equally likely”. Allow “representative” provided it’s clearly “of company” (not city) Not just “independent”. Withhold if extra wrong bits. |
| (ii) | List population, number sequentially Select using random numbers | B1 B1 [2] | List can be implied; must imply employees or people. “Sequential” can be assumed. Not “select numbers randomly”, Don’t need “ignore outside range” etc. Number randomly <i>and</i> select randomly, B1, but “assign random nos & arrange”, B2 SC: Put names into hat/lottery machine and take them out: B2 SC: Systematic: B1 for list, can get second B1 if starting-point random |
| (iii) | $H_0: p = 0.4; H_1: p < 0.4$ B(12, 0.4) | B2 M1 | Both correct, B2. Allow π . One error, e.g. μ or no symbol, B1, but \bar{x} , z etc: B0. B(12, 0.4) stated or implied. Can be implied by N(4.8, 2.88) but no further marks. 0.1673, 0.0398, 0.1513, 0.0421: M1A0(A1M1A1) |
| α : | $P(\leq 2) = 0.0834$ > 0.05 | A1 A1 | $P(\leq 2) = 0.0834$, or $P(> 2) = 0.9166$. Compare numerical $P(\leq 2)$ with 0.05, or $P(> 2)$ with 0.95 |
| β : | CR is ≤ 1 0.0196 seen and compare 2 with ≤ 1 | A1 A1 | CR is ≤ 1 stated. Explicitly compare 2 with CR, probability 0.0196 must be seen |
| | Do not reject H_0 . Insufficient evidence that proportion of employees from group Z is less. | M1 A1ft [7] | Correct first conclusion, needs $P(\leq 2 p = 0.4)$ or fully consistent equivalent In context (mention “employees”, “city” etc), acknowledge uncertainty (“evidence”) <i>Not</i> “there is evidence that the proportion of employees is 0.4” FT on wrong p -value or wrong critical value if previous mark gained SC: Normal: B2 M1 max SC: $P(= 2)$ or $P(\geq 2)$ or $P(< 2)$: B2 M1 max SC: two-tailed: can get B1B0 M1A1A0 M1A1 (don’t give second A1 for 0.05) |
| (iv) | Yes as H_0 is rejected | M1 A1 [2] | Realise this changes conclusion (FT!), or “more likely to reject H_0 ”, “larger CR” More supportive [just “more supportive” without evidence is M0A0] |

Q8, (Jun 2016, Q5)

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| (i) | $H_0: p = 0.55, H_1: p > 0.55$ $R \sim B(8, 0.55)$ where R is the number of girls $\alpha: P(R \geq 6) = 1 - 0.7799 = 0.2201 > 0.1$ | B2 M1 A1 B1 | All correct, B2. One error (e.g. \neq , wrong or no letter) B1, but r, x etc: B0 $B(8, 0.55)$ stated or implied, e.g. $N(4.4, 1.98)$ $P(\geq 6) = 0.2201$, or $P(< 6) = 0.7799$ Compare $P(\geq 6)$ with 0.1 or $P(< 6)$ with 0.9 |
| | $\beta: CR \text{ is } \geq 7 \text{ and } 6 < 7$ $p = 0.0632$ | B1 A1 | Correct CR stated and explicit comparison with 6 This probability seen, a.r.t. 0.0632. Award if 0.9368 seen and CR is correct. If CR not clearly stated, cannot get last M1A1 |
| | Do not reject H_0 . There is insufficient evidence that the girls are proportionately more likely to become Head Student. | M1 A1 7 | Correct first conclusion, requires $B(8, 0.55)$, <i>not</i> $P(> 6) [= 0.0632]$ or $P(\leq 6) [= 0.9368]$ or $P(= 6) [= 0.1569]$. Allow 0.7799 if compared with 0.9 Interpreted, in context, acknowledge uncertainty, double negative. SC: Normal: max B2 M1 SC: Two different attempts: max B2 M1 unless both correct |
| (ii) | Assume that the last 8 years are a random sample of years when Head Student has been chosen | B1 1 | Refer to random sample, allow implied by any method described Must be choosing <i>years</i> , not <i>students</i> <i>Not</i> quote conditions for random sample unless explicitly “years” Extras: ignore unless clearly wrong, in which case B0 |