Trigonometric Equations and Identities Exam Questions MS (From OCR 4722)

Q1, (Jun 2012, Q7a)

(a)	(i)	$\cos\alpha = {}^{5}/_{\sqrt{29}}$	M1	Attempt cos α
			A1	Obtain ${}^{5}/_{\sqrt{29}}$
			[2]	
(a)	(ii)	$\cos\beta = {}^{-\sqrt{40}}/_{7}$	M1	Attempt cos β
			A1	Obtain ${}^{\sqrt{40}}/_{7}$
			A1 FT	Obtain ${}^{-\sqrt{40}}/_{7}$, or –ve of their exact numerical value for cos β
			[3]	

Q2, (OCR 4752, Jun 2006, Q3)

1/√15 i.s.w. not +/–	3	M2 for √15 seen M1 for rt angled triangle with side 1 and hyp 4, or $\cos^2\theta = 1 - 1/4^2$.	3

Q3, (OCR 4752, Jan 2007, Q3)

√8 or 2√2 not ±√8	3	M1 for use of $\sin^2\theta + (1/3)^2 = 1$ and M1for sinθ = √8/3 (ignore ±) Diag.: hypot = 3, one side =1 M1 3rd side √8 M1	3

Q4, (OCR 4752, Jan 2008, Q3)

right angled triangle with 1 and 2 on correct sides Pythagoras used to obtain hyp = √5 $\cos\theta = \frac{a}{h} = \frac{2}{\sqrt{5}}$	M1 M1 A1	or M1 for sinθ = ½cosθ and M1 for substituting in $\sin^2\theta + \cos^2\theta = 1$ E1 for sufficient working	3

Q5, (Jan 2010, Q1)

(i)	$2(1-\cos^2 x) = 5\cos x - 1$ $2\cos^2 x + 5\cos x - 3 = 0$ **A.G.**	M1 A1	 2	Use $\sin^2 x = 1 - \cos^2 x$ Show given equation correctly
(ii)	$(2\cos x - 1)(\cos x + 3) = 0$ $\cos x = ½$ $x = 60°$ $x = 300°$	M1 M1 A1 A1√	 4	Recognise equation as quadratic in cos x and attempt recognisable method to solve Attempt to find x from root(s) of quadratic Obtain 60° or $\pi/3$ or 1.05 rad Obtain 300° only (or 360° – their x) and no extra in range **SR** answer only is B1 B1
			6	

Q6, (Jun 2010, Q7)

(i)	$\frac{\sin^2 x - \cos^2 x}{1-\sin^2 x} = \frac{\sin^2 x - \cos^2 x}{\cos^2 x}$ $= \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}$ $= \tan^2 x - 1$	**M1** **A1**	 2	Use either $\sin^2 x + \cos^2 x = 1$, or $\tan x = \sin x / \cos x$ Use other identity to obtain given answer convincingly.
(ii)	$\tan^2 x - 1 = 5 - \tan x$ $\tan^2 x + \tan x - 6 = 0$ $(\tan x - 2)(\tan x + 3) = 0$ $\tan x = 2$, $\tan x = -3$ $x = 63.4°, 243°$ $x = 108°, 288°$	**B1** **M1** **A1** **M1** **A1ft** **A1**	 6	State correct equation Attempt to solve three term quadratic in $\tan x$ Obtain 2 and -3 as roots of their quadratic Attempt to solve $\tan x = k$ (at least one root) Obtain at least 2 correct roots Obtain all 4 correct roots
			8	

Q7, (Jun 2013, Q2)

(i)		$\frac{1}{2}x = 53.1°, 126.9°$	B1	Obtain 106°, or better
		$x = 106°, 254°$	M1	Attempt correct solution method to find second angle
			A1	Obtain 254°, or better
			[3]	
(ii)		$\tan x = 3$	B1	State $\tan x = 3$
		$x = 71.6°, 252°$	M1	Attempt to solve $\tan x = k$
			A1	Obtain 71.6° and 252°, or better
			[3]	

Q8, (Jun 2009, Q5)

5	**(i)**	$2x = 30°, 150°$	M1		Attempt $\sin^{-1} 0.5$, then divide or multiply by 2
		$x = 15°, 75°$	A1		Obtain 15° (allow $\pi/_{12}$ or 0.262)
			A1	**3**	Obtain 75° (not radians), and no extra solutions in range
	(ii)	$2(1 - \cos^2 x) = 2 - \sqrt{3}\cos x$	M1		Use $\sin^2 x = 1 - \cos^2 x$
		$2\cos^2 x - \sqrt{3}\cos x = 0$	A1		Obtain $2\cos^2 x - \sqrt{3}\cos x = 0$ or equiv (no constant terms)
		$\cos x\,(2\cos x - \sqrt{3}) = 0$	M1		Attempt to solve quadratic in $\cos x$
		$\cos x = 0$, $\cos x = \frac{1}{2}\sqrt{3}$	A1		Obtain 30° (allow $\pi/_6$ or 0524), and no extra solns in range
		$x = 90°$, $x = 30°$	B1	**5**	Obtain 90° (allow $\pi/_2$ or 1.57), from correct quadratic only

SR answer only B1 one correct solution
B1 second correct solution, and no others

8

Q9, (Jun 2014, Q4)

(i)		$\tan x(\sin x - \cos x) = 6\cos x$ $\tan x\,(^{\sin x}/_{\cos x} - 1) = 6$ $\tan x(\tan x - 1) = 6$	M1	Use $\tan x = \frac{\sin x}{\cos x}$ correctly once
		$\tan^2 x - \tan x = 6$ $\tan^2 x - \tan x - 6 = 0$ **AG**	A1	Obtain $\tan^2 x - \tan x - 6 = 0$
			[2]	
(ii)		$(\tan x - 3)(\tan x + 2) = 0$ $\tan x = 3$, $\tan x = -2$	M1	Attempt to solve quadratic in tan x
		$x = \tan^{-1}(3)$, $x = \tan^{-1}(-2)$	M1	Attempt to solve $\tan x = k$ at least once
		$x = 71.6°, 252°, 117°, 297°$	A1	Obtain two correct solutions
			A1	Obtain all 4 correct solutions, and no others in range

Q10, (Jun 2017, Q9) [Modified]

(i)		$f(\frac{1}{2}) = \frac{1}{2} + \frac{9}{2} - 5 = 0$	B1	Confirm $f(\frac{1}{2}) = 0$, with detail shown
		$f(x) = (2x-1)(2x^2+x+5)$	M1	Attempt complete division or equiv
			A1	Obtain correct quotient
			A1	Obtain $(2x-1)(2x^2+x+5)$
(ii)	**(a)**	$4\sin 2\theta\cos 2\theta + \dfrac{5}{\cos 2\theta} = \dfrac{13\sin 2\theta}{\cos 2\theta}$	B1	Use $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$ or $\tan 2\theta\cos 2\theta = \sin 2\theta$
		$4\sin 2\theta\cos^2 2\theta + 5 = 13\sin 2\theta$	B1	Correct method to remove fraction(s)
		$4\sin 2\theta(1 - \sin^2 2\theta) + 5 = 13\sin 2\theta$	B1	Use $\cos^2 2\theta = 1 - \sin^2 2\theta$
		$4\sin 2\theta - 4\sin^3 2\theta + 5 = 13\sin 2\theta$		
		$4\sin^3 2\theta + 9\sin 2\theta - 5 = 0$	B1	Obtain correct equation, from correct working
			[4]	

$(2\sin 2\theta - 1)(2\sin^2 2\theta + \sin 2\theta + 5) = 0$ $\sin 2\theta = \frac{1}{2}$	B1	State that $\sin 2\theta = \frac{1}{2}$ oe
	M1	Attempt to solve $\sin 2\theta = \pm\frac{1}{2}$ to find at least one root
$2\theta =$ 30, 150, 390, 510	A1	Obtain at least 2 correct roots
$\theta =$ 15, 75, 195, 255	A1	Obtain 4 correct roots

Q11, (OCR 4752, Jun 2009, Q7)

use of $\cos^2\theta = 1 - \sin^2\theta$	M1		
at least one correct interim step in obtaining $4\sin^2\theta - \sin\theta = 0$.	M1	NB answer given	
$\theta = 0$ and 180,	B1		
14.(47…)	B1	r.o.t to nearest degree or better	
165 - 166	B1	-1 for extras in range	5

Q12, (OCR 4752, Jun 2011, Q7)

$\frac{\sin\theta}{\cos\theta} = 2\sin\theta$	**M1**	*may* be implied by $2\cos\theta - 1 = 0$ or better	or, if to advantage of candidate
$2\cos\theta - 1 = 0$ *and* $\sin\theta = 0$	**A1**		**B4** for all 5 correct **B3** for 4 correct
$[\theta =]$ 0, 180, 360,	**B1**		**B2** for 3 correct
$[\theta =]$ 60, 300	**B1**		**B1** for 2 correct
if 4 marks awarded, lose 1 mark for extra values in the range, ignore extra values outside the range			if extra value(s) in range, deduct one mark from total do not award if values embedded in trial and improvement approach

Q13, (Jan 2008, Q9)

(i) $(90^\circ, 2), (-90^\circ, -2)$	B1		State at least 2 correct values
	B1	**2**	State all 4 correct values (radians is B1 B0)
(ii) **(a)** $180 - \alpha$	B1	**1**	State $180 - \alpha$
(b) $-\alpha$ or $\alpha - 180$	B1	**1**	State $-\alpha$ or $\alpha - 180$ (radians or unsimplified is B1B0)
(iii) $2\sin x = 2 - 3\cos^2 x$ $2\sin x = 2 - 3(1 - \sin^2 x)$	M1		Attempt use of $\cos^2 x = 1 - \sin^2 x$
$3\sin^2 x - 2\sin x - 1 = 0$	A1		Obtain $3\sin^2 x - 2\sin x - 1 = 0$ aef with no brackets
$(3\sin x + 1)(\sin x - 1) = 0$	M1		Attempt to solve 3 term quadratic in $\sin x$
$\sin x = -{}^1/_3$, $\sin x = 1$	A1		Obtain $x = -19.5^\circ$
$x = -19.5^\circ, -161^\circ, 90^\circ$	A1√		Obtain second correct answer in range, following their x
	A1	**6**	Obtain 90° (radians or extra answers is max 5 out of 6)
			SR: answer only (and no extras) is B1 B1√ B1
		10	

Q14, (OCR 4752, Jun 2013, Q9)

(i)		$\left(\dfrac{\frac{\sin\theta}{\cos\theta}}{\cos\theta}\right) = 1$ oe	M1		
		$\sin\theta = \cos^2\theta$ and completion to given result	A1	www	
			[2]		
(ii)		$\sin^2\theta + \sin\theta - 1[=0]$	M1	allow 1 on RHS if attempt to complete square	condone $y^2 + y - 1 = 0$ mark to benefit of candidate
		$[\sin\theta =]\dfrac{-1\pm\sqrt{5}}{2}$ oe may be implied by correct answers	A1	may be implied by correct answers	ignore any work with negative root & condone omission of negative root with no comment eg **M1** for 0.618…
		$[\theta =]$ 38.17… ,or 38.2 and 141.83…, 141.8 or 142	A1	ignore extra values outside range, **A0** if extra values in range or in radians **NB** 0.6662 and 2.4754 if working in radian mode earns **M1A1A0**	if unsupported, **B1** for one of these, **B2** for both. If both values correct with extra values in range, then **B1**. **NB** 0.6662 and 2.4754 to 3sf or more
			[3]		

Q15, (Jun 2014, Q4)

(i)	$\tan x(\sin x - \cos x) = 6\cos x$ $\tan x(^{\sin x}/_{\cos x} - 1) = 6$ $\tan x(\tan x - 1) = 6$ $\tan^2 x - \tan x = 6$ $\tan^2 x - \tan x - 6 = 0$ **AG**	M1 A1 **[2]**	Use $\tan x = \frac{\sin x}{\cos x}$ correctly once Obtain $\tan^2 x - \tan x - 6 = 0$	Must be used clearly at least once - either explicitly or by writing eg 'divide by $\cos x$' at side of solution Allow M1 for any equiv eg $\sin x = \cos x \tan x$ Allow poor notation eg writing just tan rather than $\tan x$ Correct equation in given form, including $= 0$ Correct notation throughout so A0 if eg tan rather than $\tan x$ seen in solution
(ii)	$(\tan x - 3)(\tan x + 2) = 0$ $\tan x = 3, \tan x = -2$	M1	Attempt to solve quadratic in $\tan x$	This M mark is just for solving a 3 term quadratic (see guidance sheet for acceptable methods) Condone any substitution used, inc $x = \tan x$
	$x = \tan^{-1}(3),\ x = \tan^{-1}(-2)$	M1	Attempt to solve $\tan x = k$ at least once	Attempt $\tan^{-1} k$ at least once Not dependent on previous mark so M0M1 possible If going straight from $\tan x = k$ to $x = \ldots$, then award M1 only if their angle is consistent with their k
	$x = 71.6°, 252°, 117°, 297°$	A1	Obtain two correct solutions	Allow 3sf or better Must come from a correct method to solve the quadratic (as far as correct factorisation or substitution into formula) Allow radian equivs ie 1.25 / 4.39 / 2.03 / 5.18
		A1 **[4]**	Obtain all 4 correct solutions, and no others in range	Must now all be in degrees Allow 3sf or better A0 if other incorrect solutions in range $0° - 360°$ (but ignore any outside this range) **SR** If no working shown then allow **B1** for each correct solution (max of **B3** if in radians, or if extra solns in range).