Polynomial Division and Factor Theorem Exam Questions (from OCR 4722)

Note: Some of these questions were designed to be attempted using the Remainder Theorem which is no longer on the A-Level maths specification. These question can still be attempted using polynomial division.

Q1, (Jan 2006, Q8i)

The cubic polynomial $2x^3 + kx^2 - x + 6$ is denoted by $f(x)$. It is given that $(x + 1)$ is a factor of $f(x)$.

(i) Show that $k = -5$, and factorise $f(x)$ completely. [6]

Q2, (Jan 2007, Q8)

The polynomial $f(x)$ is defined by $f(x) = x^3 - 9x^2 + 7x + 33$.

(i) Find the remainder when $f(x)$ is divided by $(x + 2)$. [2]

(ii) Show that $(x - 3)$ is a factor of $f(x)$. [1]

(iii) Solve the equation $f(x) = 0$, giving each root in an exact form as simply as possible. [6]

(OCR 4722, Jun 2007, Q9i)

The polynomial $f(x)$ is given by

$$f(x) = x^3 + 6x^2 + x - 4.$$

(i) (a) Show that $(x + 1)$ is a factor of $f(x)$. [1]

(b) Hence find the exact roots of the equation $f(x) = 0$. [6]

Q4 (Jun 2008, Q4)

The cubic polynomial $ax^3 - 4x^2 - 7ax + 12$ is denoted by $f(x)$.

(i) Given that $(x - 3)$ is a factor of $f(x)$, find the value of the constant a. [3]

(ii) Using this value of a, find the remainder when $f(x)$ is divided by $(x + 2)$. [2]

Q5 (Jun 2009, Q7)

The polynomial $f(x)$ is given by $f(x) = 2x^3 + 9x^2 + 11x - 8$.

(i) Find the remainder when $f(x)$ is divided by $(x + 2)$. **[2]**

(ii) Use the factor theorem to show that $(2x - 1)$ is a factor of $f(x)$. **[2]**

(iii) Express $f(x)$ as a product of a linear factor and a quadratic factor. **[3]**

(iv) State the number of real roots of the equation $f(x) = 0$, giving a reason for your answer. **[2]**

Q6 (Jan 2012, Q5)

The cubic polynomial f(x) is defined by f(x) = $2x^3 + 3x^2 - 17x + 6$.

(i) Find the remainder when f(x) is divided by ($x - 3$). **[2]**

(ii) Given that f(2) = 0, express f(x) as the product of a linear factor and a quadratic factor. **[4]**

(iii) Determine the number of real roots of the equation f(x) = 0, giving a reason for your answer. **[2]**

360.

Q7 (Jun 2013, Q9)

The cubic polynomial f(x) is defined by f(x) = $4x^3 - 7x - 3$.

(i) Find the remainder when f(x) is divided by ($x - 2$). **[2]**

(ii) Show that ($2x + 1$) is a factor of f(x) and hence factorise f(x) completely. **[6]**

(iii) Solve the equation

$$4\cos^3\theta - 7\cos\theta - 3 = 0$$

for $0 \leqslant \theta \leqslant 360$. Give each solution for θ in an exact form. **[4]**

Q8 (Jun 2014, Q7)

The cubic polynomial f(x) is defined by f(x) = $12 - 22x + 9x^2 - x^3$.

(i) Find the remainder when f(x) is divided by ($x + 2$). **[2]**

(ii) Show that ($3 - x$) is a factor of f(x). **[1]**

(iii) Express f(x) as the product of a linear factor and a quadratic factor. **[3]**

(iv) Hence solve the equation f(x) = 0, giving each root in simplified surd form where appropriate. **[3]**
