

Integration Exam Questions MS (From OCR 4722 unless otherwise stated)

Q1, (Jun 2007, Q6)

(a) (i) $\int x^3 - 4x = \frac{1}{4}x^4 - 2x^2 + c$

M1 | Expand and attempt integration
 A1 | Obtain $\frac{1}{4}x^4 - 2x^2$ (A0 if \int or dx still present)
 B1 3 | + c (mark can be given in (b) if not gained here)

(ii) $\left[\frac{1}{4}x^4 - 2x^2\right]_1^6$
 $= (324 - 72) - (\frac{1}{4} - 2)$
 $= 253\frac{3}{4}$

M1 | Use limits correctly in integration attempt (ie F(6) - F(1))
 A1 2 | Obtain $253\frac{3}{4}$ (answer only is M0A0)

(b) $\int 6x^{-3} dx = -3x^{-2} + c$

B1 | Use of $\frac{1}{x^3} = x^{-3}$
 M1 | Obtain integral of the form kx^{-2}
 A1 3 | Obtain correct $-3x^{-2} (+c)$
 (A0 if \int or dx still present, but only penalise once in question)

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Q2, (Jan 2008, Q7)

(i) Some of the area is below the x-axis

B1 1 | Refer to area / curve below x-axis or 'negative area' ...

(ii)

M1 | Attempt integration with any one term correct
 A1 | Obtain $\frac{1}{3}x^3 - \frac{3}{2}x^2$

$\left[\frac{1}{3}x^3 - \frac{3}{2}x^2\right]_0^3 = (9 - \frac{27}{2}) - (0 - 0)$
 $= -4\frac{1}{2}$

M1 | Use limits 3 (and 0) - correct order / subtraction
 A1 | Obtain $(-4)\frac{1}{2}$

$\left[\frac{1}{3}x^3 - \frac{3}{2}x^2\right]_3^5 = (\frac{125}{3} - \frac{75}{2}) - (9 - \frac{27}{2})$
 $= 8\frac{2}{3}$

M1 | Use limits 5 and 3 - correct order / subtraction
 A1 | Obtain $8\frac{2}{3}$ (allow 8.7 or better)

Hence total area is $13\frac{1}{6}$

A1 7 | Obtain total area as $13\frac{1}{6}$, or exact equiv
 SR: if no longer $\int f(x)dx$, then B1 for using [0, 3] and [3, 5]

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Q3, (Jan 2009, Q1)

(i) $\int (x^3 + 8x - 5) dx = \frac{1}{4}x^4 + 4x^2 - 5x + c$

M1 | Attempt integration - increase in power for at least 2 terms
 A1 | Obtain at least 2 correct terms
 A1 3 | Obtain $\frac{1}{4}x^4 + 4x^2 - 5x + c$ (and no integral sign or dx)

(ii) $\int 12x^{\frac{1}{2}} dx = 8x^{\frac{3}{2}} + c$

B1 | State or imply $\sqrt{x} = x^{\frac{1}{2}}$
 M1 | Obtain $kx^{\frac{3}{2}}$
 A1 3 | Obtain $8x^{\frac{3}{2}} + c$ (and no integral sign or dx)
 (only penalise lack of + c, or integral sign or dx once)

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Q4, (Jun 2011, Q2)

(i) $\int (6x^{\frac{1}{2}} - 1) dx = 4x^{\frac{3}{2}} - x + c$

M1 Obtain $kx^{\frac{3}{2}}$

A1 Obtain $4x^{\frac{3}{2}}$

B1 3 Obtain $-x$ (don't penalise lack of $+c$)

(ii) $y = 4x^{\frac{3}{2}} - x + c$
 $17 = 32 - 4 + c \Rightarrow c = -11$
 hence $y = 4x^{\frac{3}{2}} - x - 11$

M1* State or imply $y =$ their integral from (i)

M1d* Attempt to find c using (4, 17)

A1 3 Obtain $y = 4x^{\frac{3}{2}} - x - 11$

Q5, (Jun 2015, Q5)

$\frac{dy}{dx} = 6x^{0.5} + c$	M1*	Attempt integration
	A1	Obtain $6x^{0.5}$ (allow no $+ c$)
$5 = 12 + c$	M1d*	Attempt to use $x = 4$, gradient = 5
$c = -7$	A1	Rearrange to obtain $c = -7$
$y = 4x^{1.5} - 7x + k$	M1 dd*	Attempt second integration
$1 = 32 - 28 + k$, hence $k = -3$	M1 ddd*	Attempt to find k using $(4, 1)$
$y = 4x^{1.5} - 7x - 3$	A1	Obtain $y = 4x^{1.5} - 7x - 3$
	[7]	

Q6, (Jan 2009, Q4)

$$4 \int_{-2}^2 (x^4 + 3) dx = \left[\frac{1}{5}x^5 + 3x \right]_{-2}^2$$

$$= \left(\frac{32}{5} + 6 \right) - \left(-\frac{32}{5} - 6 \right)$$

$$= 24 \frac{4}{5}$$

area of rectangle = 19×4

hence shaded area = $76 - 24 \frac{4}{5}$

$$= 51 \frac{1}{5}$$

OR

$$\begin{aligned} \text{Area} &= 19 - (x^4 + 3) \\ &= 16 - x^4 \end{aligned}$$

$$\int_{-2}^2 (16 - x^4) dx = \left[16x - \frac{1}{5}x^5 \right]_{-2}^2$$

$$= \left(32 - \frac{32}{5} \right) - \left(-32 - \frac{-32}{5} \right)$$

$$= 51 \frac{1}{5}$$

M1 Attempt integration – increase of power for at least 1 term

A1 Obtain correct $\frac{1}{5}x^5 + 3x$

M1 Use limits (any two of -2, 0, 2), correct order/subtraction

A1 Obtain $24 \frac{4}{5}$

B1 State or imply correct area of rectangle

M1 Attempt correct method for shaded area

A1 7 Obtain $51 \frac{1}{5}$ aef such as $51.2, \frac{256}{5}$

M1 Attempt subtraction, either order

A1 Obtain $16 - x^4$ (not from $x^4 + 3 = 19$)

M1 Attempt integration

A1 Obtain $\pm(16x - \frac{1}{5}x^5)$

M1 Use limits – correct order / subtraction

A1 Obtain $\pm 51 \frac{1}{5}$

A1 Obtain $51 \frac{1}{5}$ only, no wrong working

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Q7, (OCR H230/01, Practice Papers Set 1, Q6)

(i)	$\frac{x^4}{4} - \frac{x^3}{3} - x^2$ $\frac{x^4}{4} - \frac{x^3}{3} - x^2 + c$	M1	1.1	Increase at least two indices by 1
		A1	1.1	At least 2 terms correct
		A1	1.2	All correct with + c
		[3]		
(ii)	DR $x^3 - x^2 - 2x = 0$ $x = 0$ or 2 (or -1) $\int_0^2 (x^3 - x^2 - 2x) dx$ $(= -\frac{8}{3})$ Hence area = $\frac{8}{3}$	M1	1.1a	Allow just $x = 0$ or 2
		A1	1.1	
		M1	1.1	
		A1	2.2a	
		[4]		

Q8, (Jan 2011, Q9)

(i) $f(3) = -108 + 81 + 30 - 3 = 0$
 hence $(x - 3)$ is a factor

B1 Show that $f(3) = 0$, detail required

B1 **2** State $(x - 3)$ as factor
 (allow $(3 - x)$ as the factor)

(ii) $f(x) = (x - 3)(-4x^2 - 3x + 1)$
 or
 $f(x) = (3 - x)(4x^2 + 3x - 1)$

M1 Attempt complete division by $(x - 3)$, or
 equiv
 (allow division by $(3 - x)$)

or

$f(x) = (x + 1)(-4x^2 + 13x - 3)$
 or
 $f(x) = (-x - 1)(4x^2 - 13x + 3)$

A1 Obtain $-4x^2 - 3x + c$ or $-4x^2 + bx + 1$
 (or the negative of these if dividing by
 $(3 - x)$)

or

$f(x) = (1 - 4x)(x^2 - 2x - 3)$
 or
 $f(x) = (4x - 1)(-x^2 + 2x + 3)$

A1 **3** Obtain $(x - 3)(-4x^2 - 3x + 1)$
 (or $(3 - x)(4x^2 + 3x - 1)$)

iii) $-4x^2 - 3x + 1 = 0$
 $(1 - 4x)(x + 1) = 0$
 $x = \frac{1}{4}, x = -1$

M1 Attempt to solve quadratic

A1 **2** Obtain $(\frac{1}{4}, 0), (-1, 0)$

(iv) $\int f(x)dx = -x^4 + 3x^3 + 5x^2 - 3x$

$$F(3) - F(1/4) = (36) - (-101/256) = 36^{101/256}$$

$$F(1/4) - F(-1) = (-101/256) - (4) = -4^{101/256}$$

Hence area = $36^{101/256} + 4^{101/256} = 40^{101/128}$

B1 Obtain $-x^4 + 3x^3 + 5x^2 - 3x$

M1* Attempt $F(3) - F(1/4)$
or $F(1/4) - F(-1)$

A1 Obtain at least one correct area, including decimal equivs

M1d* Attempt full method to find total area including dealing correctly with negative area

A1 5 Obtain $40^{101/128}$ or $5221/128$ or 40.8

Q9, (Jun 2013, Q7)

<p>(i)</p>	$\int_1^4 (x^{\frac{3}{2}} - 1) dx = \left[\frac{2}{5} x^{\frac{5}{2}} - x \right]_1^4$ $= (12.8 - 4) - (0.4 - 1)$ $= 9^{2/5} \text{ AG}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>Attempt integration</p> <p>Obtain fully correct integral</p> <p>Attempt correct use of limits</p> <p>Obtain $9^{2/5}$</p>	
	<p>(ii)</p>	$m = \frac{3}{2} \times \sqrt{4} = 3$ $y = 3x - 5$ <p>tangent crosses x-axis at $(\frac{5}{3}, 0)$</p> $\text{area of triangle} = \frac{1}{2} \times (4 - \frac{5}{3}) \times 7$ $= 8^{1/6}$ $\text{shaded area} = 9^{2/5} - 8^{1/6} = 1^{7/30}$	<p>M1*</p> <p>M1d*</p> <p>A1</p> <p>M1d**</p> <p>A1</p> <p>[5]</p>	<p>Attempt to find gradient at (4, 7) using differentiation</p> <p>Attempt to find point of intersection of tangent with x-axis or attempt to find base of triangle</p> <p>Obtain $x = \frac{5}{3}$ as pt of intersection or obtain $\frac{7}{3}$ as base of triangle</p> <p>Attempt complete method to find shaded area</p> <p>Obtain $1^{7/30}$, or exact equiv</p>