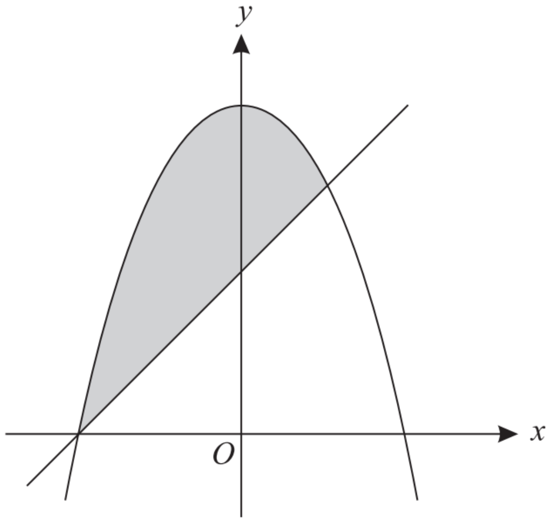
Area Between a Curve and the x-Axis

Q1, (Jun 2010, Q6a)

Use integration to find the exact area of the region enclosed by the curve $y = x^2 + 4x$, the x-axis and the lines $x = 3$ and $x = 5$. **[4]**

Q2, (Jun 2006, Q4)



The diagram shows the curve $y = 4 - x^2$ and the line $y = x + 2$.

(i) Find the x-coordinates of the points of intersection of the curve and the line. [2]

(ii) Use integration to find the area of the shaded region bounded by the line and the curve. [6]

Q3, (Jan 2006, Q8)

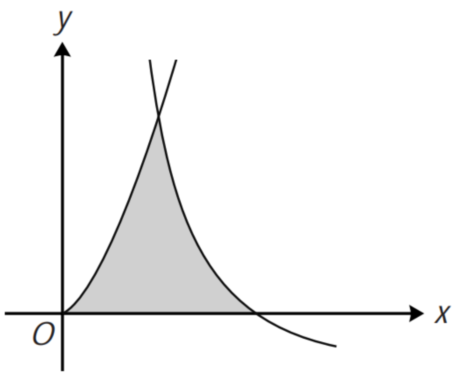
The cubic polynomial $2x^3 + kx^2 - x + 6$ is denoted by $f(x)$. It is given that $(x + 1)$ is a factor of $f(x)$.

(i) Show that $k = -5$, and factorise $f(x)$ completely. [6]

(ii) Find $\int_{-1}^{2} f(x)\,dx$. [4]

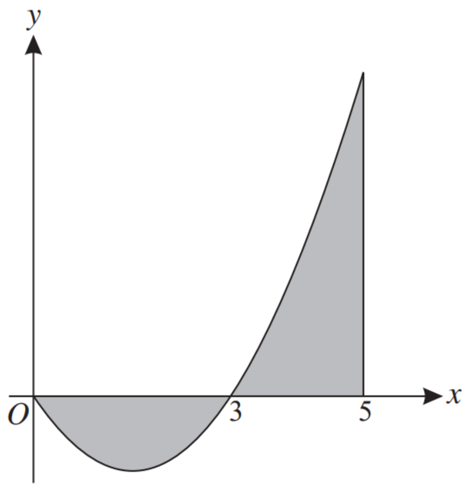
(iii) Explain with the aid of a sketch why the answer to part **(ii)** does not give the area of the region between the curve $y = f(x)$ and the x-axis for $-1 \leqslant x \leqslant 2$. [2]

Q4, (Jan 2012, Q7b)



The diagram shows the curve $y = 6x^{\frac{3}{2}}$ and part of the curve $y = \frac{8}{x^2} - 2$, which intersect at the point (1, 6). Use integration to find the area of the shaded region enclosed by the two curves and the x-axis. **[8]**

Q5, (Jan 2008, Q7)

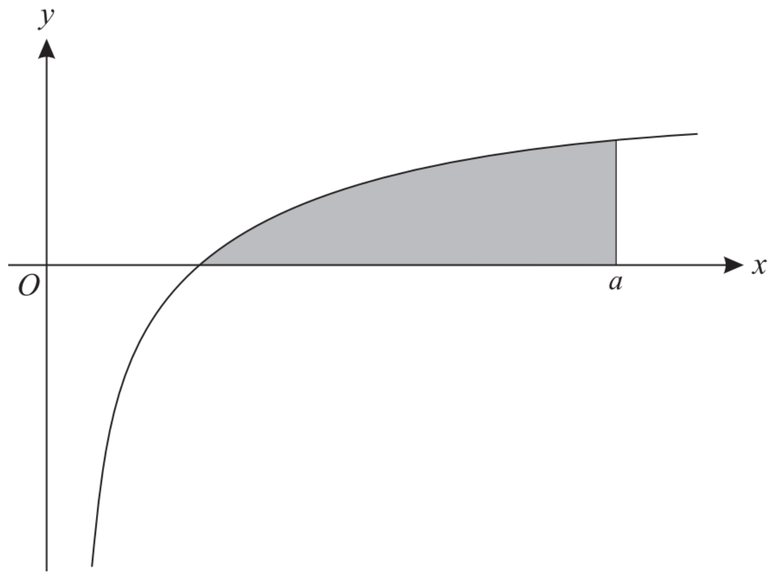


The diagram shows part of the curve $y = x^2 - 3x$ and the line $x = 5$.

(i) Explain why $\int_0^5 (x^2 - 3x)\,dx$ does not give the total area of the regions shaded in the diagram. [1]

(ii) Use integration to find the exact total area of the shaded regions. [7]

Q6, (Jan 2007, Q10)

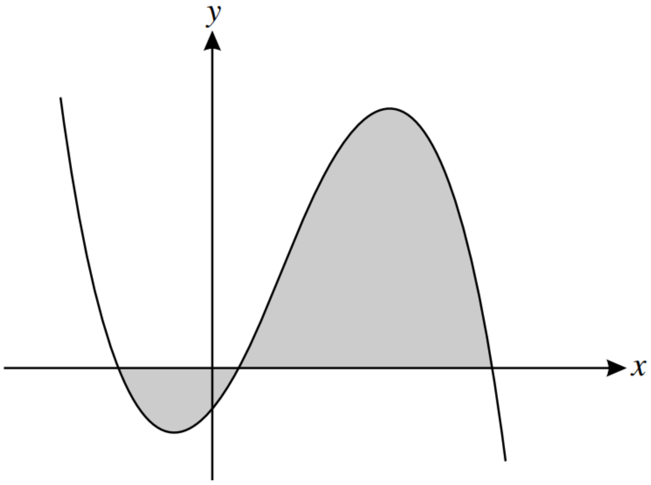


The diagram shows the graph of $y = 1 - 3x^{-\frac{1}{2}}$.

(i) Verify that the curve intersects the x-axis at $(9, 0)$. [1]

(ii) The shaded region is enclosed by the curve, the x-axis and the line $x = a$ (where $a > 9$). Given that the area of the shaded region is 4 square units, find the value of a. [9]

Q7, (Jan 2011, Q9)



The diagram shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = -4x^3 + 9x^2 + 10x - 3$.

(i) Verify that the curve crosses the x-axis at $(3, 0)$ and hence state a factor of $\mathrm{f}(x)$. **[2]**

(ii) Express $\mathrm{f}(x)$ as the product of a linear factor and a quadratic factor. **[3]**

(iii) Hence find the other two points of intersection of the curve with the x-axis. **[2]**

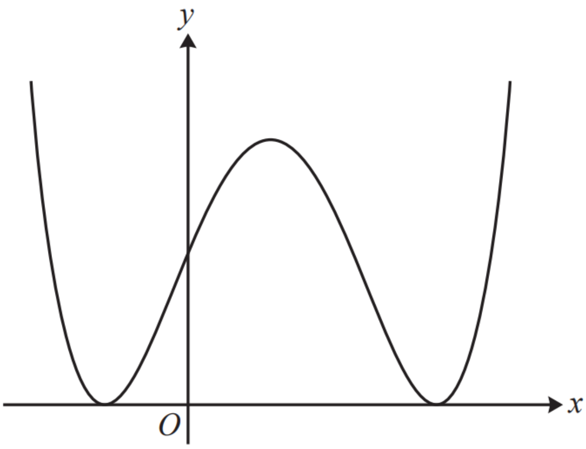
(iv) The region enclosed by the curve and the x-axis is shaded in the diagram. Use integration to find the total area of this region. **[5]**

Q8, (Jan 2016, Q7)

The cubic polynomial f(x) is defined by $\mathrm{f}(x) = x^3 - 3x^2 - x + 3$.

(i) Find the quotient and remainder when f(x) is divided by $(x + 1)$. **[3]**

(ii) Hence find the three roots of the equation $\mathrm{f}(x) = 0$. **[3]**



The diagram shows the curve C with equation $y = x^4 - 4x^3 - 2x^2 + 12x + 9$.

(iii) Show that the x-coordinates of the stationary points on C are given by $x^3 - 3x^2 - x + 3 = 0$. **[2]**

(iv) Use integration to find the exact area of the region enclosed by C and the x-axis. **[4]**

Q9, (Jan 2015, Q6)

The cubic polynomial f(x) is defined by $f(x) = x^3 - 19x + 30$.

(i) Given that $x = 2$ is a root of the equation $f(x) = 0$, express f(x) as the product of 3 linear factors. **[4]**

(ii) Use integration to find the exact value of $\int_{-5}^{3} f(x)\,dx$. **[4]**

(iii) Explain with the aid of a sketch why the answer to part **(ii)** does not give the area enclosed by the curve $y = f(x)$ and the x-axis for $-5 \leqslant x \leqslant 3$. **[2]**