Normal Distribution Exam Questions

Q1, (Edexcel 6683, Jun 2005, Q6)

(a) $M \sim N(155, 3.5^2)$

$$P(M > 160) = P\left(z > \frac{160-155}{3.5}\right)$$

standardising $\pm(160-155), \sigma, \sigma^2, \sqrt{\sigma}$ — M1

$$= P(z > 1.43)$$ — A1

$$= 0.0764$$ — A1

(3)

(b) $P(150 \le M \le 157) = P(-1.43 \le z \le 0.57)$ — awrt -1.43, 0.57 — B1 B1

$= 0.7157 - (1 - 0.9236)$ — p>0.5 — M1

$= 0.6393$ — 0.6393 - 0.6400 4dp — A1

(4)

special case : answer only B0 B0 M1 A1

(c) $P(M \le m) = 0.3 \Rightarrow \frac{m-155}{3.5} = -0.5244$ — -0.5244 att stand = z value for A1 may use awrt to -0.52. — B1 M1 A1

$m = 153.2$ — cao — A1

(4)

Q2, (Edexcel 6683, Jun 2007, Q6)

(a) $P(X > 25) = P\left(Z > \frac{25-20}{4}\right)$ — **M1**

$= P(Z > 1.25)$ — **M1**

$= 1 - 0.8944$

$= 0.1056$ — **A1**

(3)

(b) $P(X < 20) = 0.5$ so $P(X < d) = 0.5 + 0.4641 = 0.9641$ — **B1**

$P(Z < z) = 0.9641, \; z = 1.80$ — **B1**

$\frac{d-20}{4} = 1.80$ — **M1**

$d = 27.2$ — **A1**

(4)

Total 7 marks

Q3, (Edexcel 6683, Jan 2008, Q6)

(a)	200 or 200g		**B1**
			(1)
(b)	$P(190 < X < 210) = 0.6$ or $P(X < 210) = 0.8$ or $P(X > 210) = 0.2$ or diagram (o.e.)		**M1**
	Correct use of 0.8 or 0.2		**A1**
	$Z = (\pm)\frac{210-200}{\sigma}$		**M1**
	$\frac{10}{\sigma} = 0.8416$	0.8416	**B1**
	$\sigma = 11.882129....$	AWRT 11.9	**A1**
			(5)
(c)	$P(X < 180) = P\left(Z < \frac{180-200}{\sigma}\right)$		**M1**
	$= P(Z < -1.6832)$		
	$= 1 - 0.9535$		**M1**
	$= 0.0465$ or AWRT 0.046		**A1**
			(3)
			Total 9 marks

Q4, (OCR 4733, Jan 2008, Q1)

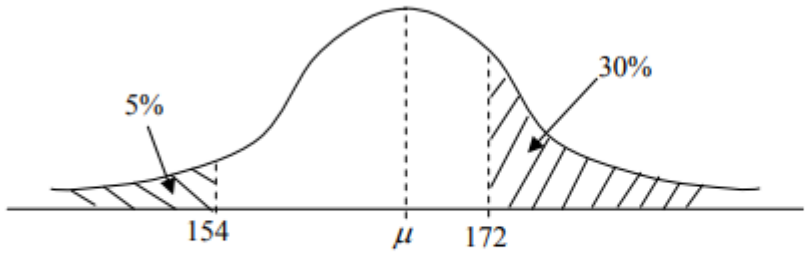
$\frac{80-\mu}{\sigma} = \Phi^{-1}(0.95) = 1.645$	M1		Standardise once with Φ^{-1}, allow σ^2, cc
	B1		Both 1.645 (1.64, 1.65) and [0.674, 0.675], ignore signs
$\frac{\mu-50}{\sigma} = \Phi^{-1}(0.75) = 0.674(5)$	A1		Both equations correct apart from wrong z, *not* 1–1.645
	M1		Solve two standardised equations
Solve simultaneously	A1		μ, a.r.t 58.7
μ = 58.7 , σ = 12.9	A1	**6**	σ, a.r.t. 12.9 [*not* σ^2] [σ^2: M1B1A0M1A1A0]

Q5, (Edexcel 6683, Jun 2008, Q7)

(a)	$z = \frac{53-50}{2}$	Attempt to standardise	**M1**
	P(X>53)=1-P(Z<1.5)	1-probability required can be implied	**B1**
	=1-0.9332		
	=0.0668		**A1**
			[3]
(b)	$P(X \le x_0) = 0.01$		**M1**
	$\frac{x_0 - 50}{2} = -2.3263$		**M1B1**
	$x_0 = 45.3474$	awrt 45.3 or 45.4	**M1A1**
			[5]
(c)	P(2 weigh more than 53kg and 1 less) $= 3 \times 0.0668^2(1 - 0.0668)$		**B1M1A1ft**
	$= 0.012492487..$	awrt 0.012	**A1**
			[4]
			Total 12

Q6, (Edexcel 6683, Jan 2010, Q7)

(a)



bell shaped, must have inflexions — B1

154,172 on axis — B1

5% and 30% — B1

(3)

(b)

$P(X < 154) = 0.05$

$\frac{154-\mu}{\sigma} = -1.6449$ or $\frac{\mu-154}{\sigma} = 1.6449$ — M1 B1

$\mu = 154 + 1.6449\sigma$ **given** — A1 cso

(3)

(c)

$172 - \mu = 0.5244\sigma$ or $\frac{172-\mu}{\sigma} = 0.5244$ (allow $z = 0.52$ or better here but must be in an equation) — B1

Solving gives $\sigma = 8.2976075$ **(awrt 8.30)** and $\mu = 167.64873$ **(awrt 168)** — M1 A1 A1

(4)

(d)

$P(\text{Taller than 160cm}) = P\left(Z > \frac{160-\mu}{\sigma}\right)$ — M1

$= P(Z < 0.9217994)$ — B1

$= 0.8212$ **awrt 0.82** — A1

(3)

Total [13]

Q7, (OCR 4733, Jun 2006, Q3)

(i)	$(140 - \mu)/\sigma = -2.326$ $(300 - \mu)/\sigma = 0.842$ Solve to obtain: μ = **257.49** σ = **50.51**	M1 B1 A1√ M1 A1 A1	**6**	One standardisation equated to Φ^{-1}, allow "1–", σ^2 Both 2.33 and 0.84 at least, ignore signs Both equations completely correct, √ on their *z* Solve two simultaneous equations to find one variable μ value, in range [257, 258] σ in range [50.4, 50.55]
(ii)	Higher as there is positive skew	B1 B1	**2**	"Higher" or equivalent stated Plausible reason, allow from normal calculations

Q8, (Edexcel 6683, Jun 2012, Q6)

(a)	$[z=]\pm\left(\frac{150-162}{7.5}\right)$		M1	
	$[z=]-1.6$		A1	
	$[\mathrm{P}(F>150)=\mathrm{P}(Z>-1.6)=]=0.9452(0071\ldots)$	**awrt <u>0.945</u>**	A1	(3)
(b)		$z=\pm 0.2533$ (or better seen)	B1	
	$(\pm)\frac{s-162}{7.5}=0.2533(47\ldots)$		M1	
	$s=163.9$	**awrt <u>164</u>**	A1	(3)
(c)		$z=\pm 1.2816$ (or better seen)	B1	
	$\frac{162-\mu}{9}=-1.2815515\ldots$		M1 A1	
	$\mu=173.533\ldots$	**awrt <u>174</u>**	A1	(4)
				[10]

Q9, (Edexcel 6683, Jun 2013(R), Q4)

(a)	$[\mathrm{P}(M<145)=]\ \mathrm{P}\left(Z<\frac{145-150}{10}\right)$		M1	
	$=\mathrm{P}(Z<-0.5)$ or $\mathrm{P}(Z>0.5)$		A1	
		= awrt <u>0.309</u>	A1	(3)
(b)	$[\mathrm{P}(B>115)=0.15\Rightarrow]\ \frac{115-100}{d}=1.0364$	(Calc gives 1.036433...)	M1B1A1	
	<u>*d* = 14.5</u>	(Calc gives 14.4727...)	A1	(4)
(c)	$[\mathrm{P}(X>\mu+15\mid X>\mu-15)=]\ \frac{\mathrm{P}(X>\mu+15)}{\mathrm{P}(X>\mu-15)}$		M1	
	$=\frac{0.35}{1-0.35}$		A1	
	$=\underline{\frac{7}{13}}$ or **<u>awrt 0.538</u>**		A1	(3)
				[10]

Q10, (Edexcel 6683, Jun 2013, Q6)

(a) [Let X be the amount of beans in a tin. $P(X < 200) = 0.1$]

$\frac{200-\mu}{7.8} = -1.2816$ [calc gives 1.28155156…] M1 B1

$\mu = 209.996\ldots$ awrt 210 A1

(3)

(b) $P(X > 225) = P\left(Z > \frac{225-"210"}{7.8}\right)$ M1

$= P(Z > 1.92)$ or $1 - P(Z < 1.92)$ (allow 1.93) A1

$= 1 - 0.9726 \quad = 0.0274$ (or better) [calc gives 0.0272037…]

$= 0.0274$

= awrt **2.7%** allow **0.027** A1

(3)

(c) [Let Y be the new amount of beans in a tin]

$\frac{210-205}{\sigma} = 2.3263$ or $\frac{200-205}{\sigma} = -2.3263$ [calc gives 2.3263478…] M1 B1

$\sigma = \frac{5}{2.3263}$ dM1

$\sigma = 2.15$ (2.14933…) A1

(4)

(10 marks)

Q11, (OCR 4733, Jan 2012, Q3)

$\mu = 60$	B1	$\mu = 60$ stated or implied, can be written down
$\frac{63.8-\mu}{\sigma} = \Phi^{-1}(0.9) = 1.282$	M1	Standardise 63.8 or 56.2 with σ, allow $\sqrt{}$ or cc errors, equate to
	B1	Φ^{-1} 1.282 (or 1.281 or 1.28) seen
$\sigma = 2.96(4)$	A1	σ, in range [2.96, 2.97], can be implied by what follows, *not* σ^2
$1-\Phi\left(\frac{65-60}{2.964}\right) = 1-\Phi(1.687)$	M1	Standardise 65 with their μ and σ, allow $\sqrt{}$ or cc errors
$= \mathbf{0.0458}$	A1	Final answer, a.r.t. 0.046, c.w.o.
	[6]	

Q12, (OCR 4733, Jun 2015, Q1)

$\frac{150-\mu}{\sigma} = 2.00$	M1	Standardise with σ, μ at least once, ignore cc, $\sqrt{}$ errors, equate to z	z not used, e.g. equated to 0.0228 and 0.9332 or 0.5092 and 0.8246: max M0M1
$\frac{143-\mu}{\sigma} = -1.5$	A1	Both LHS and signs of RHS correct	One z, one not: M1A0B0
Solve to get	B1	Both z-values correct to 3 SF	
$\boldsymbol{\mu = 146,\ \sigma = 2}$	M1	Correct method for solution	Withhold if elimination done wrongly
	A1	$\mu \in [145.95, 146.05)$ www	$\sqrt{\sigma}$ or σ^2: can get M1A0B1M1A1A0
	A1	$\sigma \in [1.995, 2.005)$ or $\sigma^2 = 4$ www	cc: M1A0B1M1A0A0
	6		

Q13, (OCR 4733, Jun 2016, Q2)

$\frac{1.03-1.00}{\sigma} = 1.645$	M1dep*	Standardise and equate to Φ^{-1}, allow wrong sign, σ^2, 1–, cc etc
	A1	All correct apart possibly from value of Φ^{-1}
	B1	1.645 seen anywhere, allow –1.645, can be implied
$[\sigma = 0.0182\ldots \approx {}^{6}/_{329}]$	*M1	Solve to find σ, or eliminate σ, dependent on first M1
$1-\Phi\left(\frac{1.05-1.03}{\sigma}\right) = 1-\Phi(1.0966)$	M1	Standardise with $\mu = 1.03$, use Φ, answer < 0.5, allow $\sqrt{}$ errors
$= 1 - 0.8635 \quad = \mathbf{0.1365}$ or 13.6(5)%	A1 **6**	Final answer in range [0.1355, 0.137] or [13.55%, 13.7%], must be from positive σ, not from σ^2
		0.1333 from $\sigma = 0.018$ is 5+A0