Standard Integral Exam Questions MS

Q1, (OCR 4723, Jan 2006, Q1)

Obtain integral of form $k \ln x$	**M1**		[any non-zero constant k; or equiv such as $k \ln 3x$]
Obtain $3 \ln 8 - 3 \ln 2$	**A1**		[or exact equiv]
Attempt use of at least one relevant log property	**M1**		[would be earned by initial $\ln x^3$]
Obtain $3 \ln 4$ or $\ln 8^3 - \ln 2^3$ and hence $\ln 64$	**A1**	**4**	[**AG**; with no errors]

Q2, (OCR 4723, Jan 2009, Q1)

(i)	Obtain integral of form ke^{-2x}	M1		any constant k different from 8
	Obtain $-4e^{-2x}$	A1		or (unsimplified) equiv
(ii)	Obtain integral of form $k(4x+5)^7$	M1		any constant k
	Obtain $\frac{1}{28}(4x+5)^7$	A1		in simplified form
	Include … $+ c$ at least once	B1		in either part
			5	

Q3, (OCR 4723, Jun 2011, Q1)

(i)	Obtain integral of form ke^{2x+1}	M1		any non-zero constant k different from 6; using substitution $u = 2x+1$ to obtain ke^u earns M1 (but answer to be in terms of x)
	Obtain correct $3e^{2x+1}$	A1		or equiv such as $\frac{6}{2}e^{2x+1}$
(ii)	Obtain integral of form $k_1 \ln(2x+1)$	M1		any non-zero constant k_1 ; allow if brackets absent; $k_1 \ln u$ (after sub'n) earns M1
	Obtain correct $5\ln(2x+1)$	A1		or equiv such as $\frac{10}{2}\ln(2x+1)$; condone brackets rather than modulus signs but brackets or modulus signs must be present (so that $5\ln 2x+1$ earns A0)
	Include … $+ c$ at least once	B1	**5**	anywhere in the whole of question 1; this mark available even if no marks awarded for integration
			5	

Q4, (OCR 4723, Jan 2006, Q5)

Obtain integral of form $k(1-2x)^6$	**M1**		[any non-zero constant k]
Obtain correct $-\frac{1}{12}(1-2x)^6$	**A1**		[or unsimplified equiv; allow $+c$]
Use limits to obtain $\frac{1}{12}$	**A1**		[or exact (unsimplified) equiv]
Obtain integral of form $k\,e^{2x-1}$	**M1**		[or equiv; any non-zero constant k]
Obtain correct $\frac{1}{2}e^{2x-1}-x$	**A1**		[or equiv; allow $+c$]
Use limits to obtain $-\frac{1}{2}e^{-1}$	**A1**		[or exact (unsimplified) equiv]
Show correct process for finding required area	**M1**		[at any stage of solution; if process involves two definite integrals, second must be negative]
Obtain $\frac{1}{12}+\frac{1}{2}e^{-1}$	**A1**	**8**	[or exact equiv; no $+c$]

Q5, (OCR 4723, Jun 2010, Q7)

Differentiate to obtain $k_1(3x-1)^3$	M1	any constant k_1
Obtain correct $12(3x-1)^3$	A1	or (unsimplified) equiv
Substitute 1 to obtain 96	A1	
Attempt to find x-coordinate of Q	M1	using tangent with $y=0$ or using gradient
Obtain $\frac{5}{6}$	A1	or exact equiv
Integrate to obtain $k_2(3x-1)^5$	M1	any constant k_2
Obtain correct $\frac{1}{15}(3x-1)^5$	A1	or (unsimplified) equiv
Use limits $\frac{1}{3}$ and 1 to obtain $\frac{32}{15}$	A1	
Attempt to find shaded area by correct process	M1	integral – triangle or equiv
Obtain ($\frac{32}{15}-\frac{1}{2}\times\frac{1}{6}\times 16$ and hence) $\frac{4}{5}$	A1	or equiv
	10	

Q6, (OCR 4723, Jun 2011, Q6)

Method 1: (Differentiation; assume value $\frac{10}{3}$; eqn of tangent; through origin)

Differentiate to obtain $k(3x-5)^{-\frac{1}{2}}$	M1	any constant k
Obtain $\frac{3}{2}(3x-5)^{-\frac{1}{2}}$	A1	or equiv
Attempt to find equation of tangent at P and attempt to show tangent passing through origin	M1	assuming value $\frac{10}{3}$; or equiv
Obtain $y=\frac{3}{2\sqrt{5}}x$ and confirm that tangent passes through O	A1	AG; necessary detail needed

Method 2: (Differentiation; equate $\frac{y\text{ change}}{x\text{ change}}$ to deriv; solve for x)

Differentiate to obtain $k(3x-5)^{-\frac{1}{2}}$	M1	any constant k
Obtain $\frac{3}{2}(3x-5)^{-\frac{1}{2}}$	A1	or equiv
Equate $\frac{y\text{ change}}{x\text{ change}}$ to deriv and attempt solution	M1	
Obtain $\frac{\sqrt{3x-5}}{x}=\frac{3}{2}(3x-5)^{-\frac{1}{2}}$ and solve to obtain $\frac{10}{3}$ only	A1	

Method 3: (Differentiation; find x from $y=\mathrm{f}'(x)\,x$ and $y=\sqrt{3x-5}$)

Differentiate to obtain $k(3x-5)^{-\frac{1}{2}}$	M1	any constant k
Obtain $\frac{3}{2}(3x-5)^{-\frac{1}{2}}$	A1	or equiv
State $y=\frac{3}{2}(3x-5)^{-\frac{1}{2}}x$, $y=\sqrt{3x-5}$, eliminate y and attempt solution	M1	condone this attempt at 'eqn of tangent'
Obtain $\frac{10}{3}$ only	A1	

Method 4: (No differentiation; general line through origin to meet curve at one point only)

Eliminate y from equations $y=kx$ and $y=\sqrt{3x-5}$ and attempt formation of quadratic eqn	M1	
Obtain $k^2x^2-3x+5=0$	A1	or equiv
Equate discriminant to zero to find k	M1	
Obtain $k=\frac{3}{2\sqrt{5}}$ or equiv and confirm $x=\frac{10}{3}$	A1	

Method 5: (No differentiation; use coords of P to find eqn of OP; confirm meets curve once)

Use coordinates $(\frac{10}{3},\sqrt{5})$ to obtain $y=\frac{3\sqrt{5}}{10}x$ or equiv as equation of OP	B1	
Eliminate y from this eqn and eqn of curve and attempt quadratic eqn	M1	should be $9x^2-60x+100=0$ or equiv
Attempt solution or attempt discriminant	M1	
Confirm $\frac{10}{3}$ only or discriminant $=0$	A1	

Either:

Integrate to obtain $k(3x-5)^{\frac{3}{2}}$	*M1		any constant k
Obtain correct $\frac{2}{9}(3x-5)^{\frac{3}{2}}$	A1		
Apply limits $\frac{5}{3}$ and $\frac{10}{3}$	M1		dep *M; the right way round
Make sound attempt at triangle area and calculate (triangle area) minus (their area under curve)	M1		or equiv
Obtain $\frac{10}{6}\sqrt{5}-\frac{10}{9}\sqrt{5}$ and hence $\frac{5}{9}\sqrt{5}$	A1	9	or exact equiv involving single term
Or:			
Arrange to $x = \ldots$ and integrate to obtain $k_1y^3+k_2y$ form	*M1		
Obtain $\frac{1}{9}y^3+\frac{5}{3}y$	A1		
Apply limits 0 and $\sqrt{5}$	M1		dep *M; the right way round
Make sound attempt at triangle area and calculate (their area from integration) minus (triangle area)	M1		
Obtain $\frac{20}{9}\sqrt{5}-\frac{5}{3}\sqrt{5}$ and hence $\frac{5}{9}\sqrt{5}$	A1	(9)	or exact equiv involving single term

9

Q7, (OCR 4723, Jun 2016, Q5)

i		Either State $e^{2x} = 8e^{-x}$ and so $e^{3x} = 8$ Obtain $e^x = 2$ and hence $x = \ln 2$	B1 B1	AG; necessary detail needed	Verifying by substitution of $\ln 2$ in each equation earns B0B0
		Or 1 State $e^{2x} = 8e^{-x}$ and so $e^{3x} = 8$ State $3x = \ln 8$, $x = \ln 8^{\frac{1}{3}}$ and hence $x = \ln 2$	B1 B1	AG; necessary detail needed	Going immediately from $x = \frac{1}{3}\ln 8$ to $x = \ln 2$ does not earn the second B1
		Or 2 State $e^{2x} = 8e^{-x}$ and $2x = \ln 8 - x$ State $3x = \ln 8$, $x = \ln 8^{\frac{1}{3}}$ and hence $x = \ln 2$	B1 B1 **[2]**	AG; necessary detail needed	Going immediately from $x = \frac{1}{3}\ln 8$ to $x = \ln 2$ does not earn the second B1
ii		Integrate to obtain $k_1 e^{-x}$ and $k_2 e^{2x}$	M1	Any non-zero constants k_1 and k_2	
		Obtain correct $-8e^{-x} - \frac{1}{2}e^{2x}$ or, if done separately, $-8e^{-x}$ and $\frac{1}{2}e^{2x}$	A1		
		Apply limits 0 and $\ln 2$ correctly to their integral(s)	M1	Condone one sign slip; earned by sight of $-8e^{-\ln 2} - \frac{1}{2}e^{2\ln 2} + 8 + \frac{1}{2}$ (or equivs if integrals treated separately)	M1 also implied by sight only of $-4 - 2 + 8 + \frac{1}{2}$ (or equivs …)
		Obtain at least $-4 - 2 + 8 + \frac{1}{2}$ (or equivs)	*A1		
		Obtain $\frac{5}{2}$ or equiv	A1 **[5]**	Final A1 dependent on *A1	

Q8, (OCR 4723, Jun 2006, Q7)

(a)	Obtain integral of form $k(4x-1)^{-1}$	**M1**		any non-zero constant k
	Obtain $-\frac{1}{2}(4x-1)^{-1}$	**A1**		or equiv; allow + c
	Substitute limits and attempt evaluation	**M1**		for any expression of form $k'(4x-1)^n$
	Obtain $\frac{2}{21}$	**A1**	**4**	or exact equiv
(b)	Integrate to obtain ln x	**B1**		
	Substitute limits to obtain ln 2a – ln a	**B1**		
	Subtract integral attempt from attempt at area of appropriate rectangle	**M1**		or equiv
	Obtain 1 – (ln 2a – ln a)	**A1**		or equiv
	Show at least one relevant logarithm property	**M1**		at any stage of solution
	Obtain 1 – ln 2 and hence $\ln(\frac{1}{2}e)$	**A1**	**6**	**AG**; full detail required