Parametric Equations (From OCR 4724)

Q1, (Jun 2005, Q7)

A curve is given parametrically by the equations

$$x = t^2, \qquad y = \frac{1}{t}.$$

(i) Find $\frac{dy}{dx}$ in terms of t, giving your answer in its simplest form. [3]

(ii) Show that the equation of the tangent at the point $P\left(4, -\frac{1}{2}\right)$ is

$$x - 16y = 12.$$ [3]

(iii) Find the value of the parameter at the point where the tangent at P meets the curve again. [4]

Q2, (Jan 2009, Q6)

A curve has parametric equations

$$x = t^2 - 6t + 4, \quad y = t - 3.$$

Find

(i) the coordinates of the point where the curve meets the x-axis, **[2]**

(ii) the equation of the curve in cartesian form, giving your answer in a simple form without brackets, **[2]**

(iii) the equation of the tangent to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[5]**

Q3, (Jun 2009, Q5)

A curve has parametric equations

$$x = 2t + t^2, \quad y = 2t^2 + t^3.$$

(i) Express $\frac{dy}{dx}$ in terms of t and find the gradient of the curve at the point $(3, -9)$. **[5]**

(ii) By considering $\frac{y}{x}$, find a cartesian equation of the curve, giving your answer in a form not involving fractions. **[4]**

Q4, (Jun 2010, Q7)

The parametric equations of a curve are $x = \frac{t+2}{t+1}$, $y = \frac{2}{t+3}$.

(i) Show that $\frac{dy}{dx} > 0$. **[6]**

(ii) Find the cartesian equation of the curve, giving your answer in a form not involving fractions. **[5]**

Q5, (Jan 2011, Q4)

A curve has parametric equations

$$x = 2 + t^2, \qquad y = 4t.$$

(i) Find $\frac{dy}{dx}$ in terms of t. **[2]**

(ii) Find the equation of the normal at the point where $t = 4$, giving your answer in the form $y = mx + c$. **[3]**

(iii) Find a cartesian equation of the curve. **[2]**

Q6, (Jun 2011, Q8)

A curve has parametric equations

$$x = \frac{1}{t+1}, \qquad y = t - 1.$$

The line $y = 3x$ intersects the curve at two points.

(i) Show that the value of t at one of these points is -2 and find the value of t at the other point. **[2]**

(ii) Find the equation of the normal to the curve at the point for which $t = -2$. **[6]**

(iii) Find the value of t at the point where this normal meets the curve again. **[2]**

(iv) Find a cartesian equation of the curve, giving your answer in the form $y = f(x)$. **[3]**

Q7, (Jan 2013, Q5)

The parametric equations of a curve are

$$x = 2 + 3\sin\theta \text{ and } y = 1 - 2\cos\theta \text{ for } 0 \leqslant \theta \leqslant \tfrac{1}{2}\pi.$$

(i) Find the coordinates of the point on the curve where the gradient is $\frac{1}{2}$. **[5]**

(ii) Find the cartesian equation of the curve. **[2]**

Q8, (Jun 2013, Q9)

A curve has parametric equations $x = \frac{1}{t} - 1$ and $y = 2t + \frac{1}{t^2}$.

(i) Find $\frac{dy}{dx}$ in terms of t, simplifying your answer. **[3]**

(ii) Find the coordinates of the stationary point and, by considering the gradient of the curve on either side of this point, determine its nature. **[4]**

(iii) Find a cartesian equation of the curve. **[2]**

Q9, (Jun 2014, Q7)

A curve has parametric equations

$$x = 2\sin t, \quad y = \cos 2t + 2\sin t$$

for $-\frac{1}{2}\pi \leqslant t \leqslant \frac{1}{2}\pi$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - 2\sin t$ and hence find the coordinates of the stationary point. **[5]**

(ii) Find the cartesian equation of the curve. **[3]**

(iii) State the set of values that x can take and hence sketch the curve. **[3]**

Q10, (Jun 2015, Q10)

(i) Express $\frac{x+8}{x(x+2)}$ in partial fractions. **[3]**

(ii) By first using division, express $\frac{7x^2+16x+16}{x(x+2)}$ in the form $P + \frac{Q}{x} + \frac{R}{x+2}$. **[3]**

A curve has parametric equations $x = \frac{2t}{1-t}$, $y = 3t + \frac{4}{t}$.

(iii) Show that the cartesian equation of the curve is $y = \frac{7x^2+16x+16}{x(x+2)}$. **[4]**

(iv) Find the area of the region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$. Give your answer in the form $L + M\ln 2 + N\ln 3$. **[4]**

Q11. (Jun 2016, Q9)

A curve has parametric equations $x = 1 - \cos t$, $y = \sin t \sin 2t$, for $0 \leqslant t \leqslant \pi$.

(i) Find the coordinates of the points where the curve meets the x-axis. **[3]**

(ii) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2\cos 2t + 2\cos^2 t$. Hence find, in an exact form, the coordinates of the stationary points. **[7]**

(iii) Find the cartesian equation of the curve. Give your answer in the form $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is a polynomial. **[3]**

(iv) Sketch the curve. **[2]**
