Integration By Substitution Exam Questions MS (From OCR 4724)

Q1, (Jun 2005, Q4)

(i) $dx = \sec^2\theta\, d\theta$ AEF	M1		Attempt to connect dx,dθ (not dx = dθ)
	A1		For dx = $\sec^2\theta\, d\theta$ or equiv correctly used
Indefinite integral = $\int \cos^2\theta\, d\theta$	A1	3	With at least one intermed step **AG**
(ii) = $k\int$ +/– 1 +/– cos 2θ dθ	M1		"Satis" attempt to change to double angle
½[θ + ½ sin 2θ]	A1		Correct attempt + correct integration
Limits = ¼π(accept 45) and 0	M1		New limits for θ or resubstituting
(π + 2)/8 AEF	A1	4	Ignore decimals after correct answer
			7
			Single 'parts' + $\sin^2\theta = 1-\cos^2\theta$ acceptable

Q2, (Jan 2006, Q6)

(i) Attempt to connect $dx, d\theta$	M1		But not $dx = d\theta$
$dx = 2\sin\theta\cos\theta\, d\theta$	A1		AEF
$\sqrt{\dfrac{x}{1-x}} = \dfrac{\sin\theta}{\cos\theta}$	B1		Ignore any references to ±.
Reduction to $\int 2\sin^2\theta\, d\theta$	A1	**4**	**AG** WWW
(ii) $\sin^2\theta = k(+/-1+/-\cos 2\theta)$	M1		Attempt to change $(2)\sin^2\theta$ into $f(\cos 2\theta)$
$2\sin^2\theta = 1-\cos 2\theta$	A1		Correct attempt
$\int \cos 2\theta\, d\theta = \frac{1}{2}\sin 2\theta$	B1		Seen anywhere in this part
Attempting to change limits	M1		<u>Or</u> Attempting to resubstitute; Accept degrees
$\frac{1}{2}\pi$	A1	**5**	
<u>Alternatively</u> Parts once & use			
$\cos^2\theta = 1-\sin^2\theta$	(M2)		Instead of the M1 A1 B1
$\frac{1}{2}(\theta - \sin\theta\cos\theta)$	(A1)		Then the final M1 A1 for use of limits

Q3, (Jan 2008, Q10i)

$(1-x^2)^{\frac{3}{2}} \to \cos^3\theta$	B1		May be implied by $\int \sec^2\theta\, d\theta$
$dx \to \cos\theta\, d\theta$	B1		
$\dfrac{1}{(1-x^2)^{\frac{3}{2}}}dx \to \sec^2\theta\,(d\theta)$ or $\dfrac{1}{\cos^2\theta}(d\theta)$	B1		
$\int \sec^2\theta\,(d\theta) = \tan\theta$	B1		
Attempt change of limits (expect 0 & $\frac{1}{6}\pi/30$)	M1		Use with $f(\theta)$; or re-subst & use 0 & $\frac{1}{2}$
$\dfrac{1}{\sqrt{3}}$ AEF	A1	**6**	Obtained with no mention of 30 anywhere

Q4, (Jun 2008, Q8)

(i)	$A(t+1)+B=2t$	**M1**	Beware: correct values for A and/or B can be ...
	$A=2$	**A1**	… obtained from a wrong identity
	$B=-2$	**A1**	Alt method: subst suitable values into given… …expressions
		3	
(ii)	Attempt to connect dx and dt	**M1**	But not just dx = dt. As **AG**, look carefully.
	dx = t dt s.o.i. AEF	**A1**	
	$x+\sqrt{2x-1}\rightarrow\frac{t^2+1}{2}+t=\frac{(t+1)^2}{2}$ s.o.i.	**B1**	Any wrong working invalidates
	$\int\frac{2t}{(t+1)^2}\,\mathrm{d}t$	**A1**	**AG** WWW The 'dt' must be present
		4	
(iii)	$\int\frac{1}{t+1}\,\mathrm{d}t=\ln(t+1)$	**B1**	Or parts $u=2t, \mathrm{d}v=(t+1)^{-2}$ or subst $u=t+1$
	$\int\frac{1}{(t+1)^2}\,\mathrm{d}t=-\frac{1}{t+1}$	**B1**	
	Attempt to change limits (expect 1 & 3) and use f(t)	**M1**	or re-substitute and use 1 and 5 on g(x)
	$\ln 4-\frac{1}{2}$	**A1**	AEF (like terms amalgamated); if A0 A0 in (i), then final A0
		4	

Q5, (Jan 2009, Q5)

(i)	Attempt to connect du and dx, find $\frac{\mathrm{d}u}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}u}$	M1	But not e.g. du = dx
	Any correct relationship, however used, such as dx = 2u du	A1	or $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2}x^{-½}$
	Subst with clear reduction (≥ 1 inter step) to **AG**	A1 **(3)**	WWW
(ii)	Attempt partial fractions	M1	
	$\frac{2}{u}-\frac{2}{1+u}$	A1	
	√ A ln u + B ln (1 + u)	√A1	Based on $\frac{A}{u}+\frac{B}{1+u}$
	Attempt integ, change limits & use on f(u)	M1	or re-subst & use 1 & 9
	$\ln\frac{9}{4}$ AEexactF (e.g. 2 ln 3 –2 ln 4 + 2 ln 2)	A1 **(5)**	Not involving ln 1
		8	

Q6, (Jan 2010, Q4)

Attempt to connect du and dt or find $\frac{du}{dt}$ or $\frac{dt}{du}$	M1	not $du = dt$ but no accuracy
$du = \frac{1}{t}dt$ or $\frac{du}{dt} = \frac{1}{t}$ or $dt = e^{u-2}du$ or $\frac{dt}{du} = e^{u-2}$	A1	
Indef int $\rightarrow \int \frac{1}{u^2}(du)$	A1	no t or dt in evidence
$= -\frac{1}{u}$	A1	
Attempt to change limits if working with $f(u)$	M1	or re-subst & use 1 and e
$\frac{1}{6}$ ISW	A1	ln e must be changed to 1, ln 1 to 0
	6	

Q7, (Jan 2011, Q5)

(i) Attempt to connect dx and du	M1		Including $\frac{du}{dx} =$ or $du = ...dx$; not $dx = du$
$5 - x = 4 - u^2$	B1		perhaps in conjunction with next line
Show $\int \frac{4-u^2}{2+u}.2u\, du$ reduced to $\int 4u - 2u^2\, du$ **AG**	A1		In a fully satisfactory & acceptable manner
Clear explanation of why limits change	B1		e.g. when $x = 2$, $u = 1$ and when $x = 5$, $u = 2$
$\frac{4}{3}$	B1	**5**	not dependent on any of first 4 marks
(ii)(a) $5 - x$	*B1	**1**	Accept $4 - x - 1 = 5 - x$ (this is not **AG**)
(b) Show reduction to $2 - \sqrt{x-1}$	dep*B1		
$\int \sqrt{x-1}\, dx = \frac{2}{3}(x-1)^{\frac{3}{2}}$	B1		Indep of other marks, seen anywhere in (b)
$\left(10 - \frac{2}{3}.8\right) - \left(4 - \frac{2}{3}\right) = \frac{4}{3}$ or $4\frac{2}{3} - 3\frac{1}{3} = \frac{4}{3}$	B1	**3**	Working must be shown
		9	

Q8, (Jan 2013, Q6)

Attempt diff to connect du & dx	M1	or find $\frac{du}{dx}$ or $\frac{dx}{du}$	
Correct result e.g. $\frac{du}{dx} = 2$ or $du = 2\,dx$	A1		
Indef integ in terms of $u = \frac{1}{2}\int \frac{2u-3}{u^5}(du)$	A1	Must be completely in terms of u.	
Integrate to $\frac{u^{-3}}{-3} - \frac{3u^{-4}}{-8}$ oe	A1A1	or (using 'by parts') $\frac{(2u-3)u^{-4}}{-8} - \frac{u^{-3}}{12}$	Award B1,B1 for $\frac{4u^{-3}}{-3} - \frac{3u^{-4}}{-2}$ or for $\frac{2u^{-3}}{-3} - \frac{3u^{-4}}{-4}$ or for $\frac{(2u-3)u^{-4}}{-2} - \frac{u^{-3}}{3}$ or for $\frac{(2u-3)u^{-4}}{-4} - \frac{u^{-3}}{6}$
Use correct variable & correct values for limits	M1	Provided minimal attempt at $\int f(u)du$ made	
$= \frac{-23}{384}$ oe $(-0.059895 \ldots..)$	A1	Accept decimal answer only if minimum of first 3 marks scored	
[ISW,e.g. changing to $\frac{23}{384}$]			
	[7]		

Q9, (Jun 2016, Q6)

$\frac{du}{dx} = 2x$ oe or $\frac{dx}{du} = \frac{1}{2}(u \pm 2)^{-\frac{1}{2}}$ oe	**M1**		
$\frac{Ax^2 + B}{2}$ or better from replacing dx NB $\frac{6x^3 + 4x}{2x} = \frac{6x^2 + 4}{2}$	**M1**		or substitution of $x = (u \pm 2)^{\frac{1}{2}}$ in denominator from $\frac{dx}{du}$
substitution of $x^2 = u \pm 2$ or $x = (u \pm 2)^{\frac{1}{2}}$ in numerator	**M1**	**NB** $3(u+2)+2$ or $3(u+2)^{\frac{3}{2}} + 2(u+2)^{\frac{1}{2}}$	
$\int (\frac{3u+8}{\sqrt{u}})[du]$ oe	**A1**	$\frac{3(u+2)+2}{\sqrt{u}}$ or better	
$\frac{3u^{\frac{3}{2}}}{\frac{3}{2}} + \frac{8u^{\frac{1}{2}}}{\frac{1}{2}}$ oe	**A1**	or $6u^{\frac{3}{2}} + 16u^{\frac{1}{2}} - 4u^{\frac{3}{2}}$ from integration by parts	
$2(x^2-2)^{\frac{3}{2}} + 16(x^2-2)^{\frac{1}{2}} + c$ cao	**A1** [6]	allow $2(x^2-2)^{\frac{1}{2}}(x^2+6) + c$ for final mark, **A0** if du not seen at some stage in the integral	must see constant of integration here or in previous line and coefficients must be simplified for final **A1**

Q10, (Jun 2017, Q9)

$\frac{\mathrm{d}u}{\mathrm{d}x} = 1 + \frac{1}{x}$	**B1**		
$x + \ln x = \pm u \pm 1$ oe substituted into the numerator	**M1***	allow slip in substitution	
$\mathrm{d}x$ replaced by *their* $\left(\frac{1}{1/x + 1}\right)[\mathrm{d}u]$ in integrand oe	**M1***		
$\int \left(\frac{3(1-(u-1))}{u}\right)[\mathrm{d}u]$ oe	**A1**	may be simplified	$\int \left(\frac{6}{u} - 3\right)\mathrm{d}u$
$A\ln u + Bu\ (+c)$	**M1dep***	following $\int \left(\frac{A}{u} + B\right)\mathrm{d}u$	
$6\ln(1 + \ln x + x) - 3(1 + \ln x + x) + c$ oe isw	**A1**		if $\mathrm{d}u$ and/or $\int$ and/or $+c$ not seen at some stage, withhold the final **A1**
	[6]		