

Implicit Differentiation Exam Questions (From OCR 4724)

Q1, (Jun 2007, Q6)

| | | |
|--|-----|--|
| (i) $\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$ | B1 | |
| Using $d(uv) = u dv + v du$ for the $(3)xy$ term | M1 | |
| $\frac{d}{dx}(x^2 + 3xy + 4y^2) = 2x + 3x \frac{dy}{dx} + 3y + 8y \frac{dy}{dx}$ | A1 | |
| Solve for $\frac{dy}{dx}$ & subst $(x, y) = (2,3)$ | M1 | or v.v. Subst now or at normal eqn stage; |
| $\frac{dy}{dx} = -\frac{13}{30}$ | A1 | (M1 dep on either/both B1 M1 earned) |
| Grad normal = $\frac{30}{13}$ follow-through | √B1 | Implied if grad normal = $\frac{30}{13}$ |
| Find equ <u>any</u> line thro $(2,3)$ with <u>any</u> num grad | M1 | This f.t. mark awarded only if numerical |
| $30x - 13y - 21 = 0$ AEF | A1 | 8 No fractions in final answer 8 |

Q2, (Jan 2008, Q6)

| | | |
|--|----|---|
| $\frac{d}{dx}(x^2y) = x^2 \frac{dy}{dx} + 2xy$ | B1 | s.o.i.; |
| $\frac{d}{dx}(y^3) = 3y^2 \frac{dy}{dx}$ | B1 | |
| Substitute $(x,y) = (1,1)$ and solve for $\frac{dy}{dx}$ | M1 | or v.v. Solve now or at normal stage. [This |
| $\frac{dy}{dx} = -\frac{11}{7}$ WWW | M1 | dep on either/both B1 earned] |
| Gradient normal = $-\frac{1}{\frac{dy}{dx}}$ | A1 | Implied if grad normal = $\frac{7}{11}$ |
| $7x - 11y + 4 = 0$ AEF | M1 | Numerical or general, awarded at any stage |
| | A1 | 6 No fractions in final answer. |

Q3, (Jan 2009, Q8)

(i) $\frac{d}{dx}(y^3) = 3y^2 \frac{dy}{dx}$ B1

Consider $\frac{d}{dx}(xy)$ as a product M1

$= x \frac{dy}{dx} + y$ A1 Tolerate omission of '6'

$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$ ISW AEF A1 (4)

(ii) $x^3 = 2^4$ or 16 and $y^3 = 2^5$ or 32 *B1

Satisfactory conclusion dep* B1 AG

Substitute $\left(2^{\frac{4}{3}}, 2^{\frac{5}{3}}\right)$ into their $\frac{dy}{dx}$ M1 or the numerator of $\frac{dy}{dx}$

Show or use calc to demo that num = 0, ignore denom AG A1 (4)

(iii) Substitute (a, a) into eqn of curve M1 & attempt to state 'a = ...'

$a = 3$ only with clear ref to $a \neq 0$ A1

Substitute $(3,3)$ or (their a , their a) into their $\frac{dy}{dx}$ M1

-1 only WWW A1 (4) from (their a , their a)

Q4, (Jun 2009, Q8)

- (i) $\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$ B1
- $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$ used on $(-7)xy$ M1
- $\frac{d}{dx}(14x^2 - 7xy + y^2) = 28x - 7x \frac{dy}{dx} - 7y + 2y \frac{dy}{dx}$ A1 (= 0)
- $2y \frac{dy}{dx} - 7x \frac{dy}{dx} = 7y - 28x \rightarrow \frac{dy}{dx} = \frac{28x-7y}{7x-2y}$ www AG A1 4 As AG, intermed step nec

- (ii) Subst $x = 1$ into eqn curve & solve quadratic eqn in y M1 ($y = 3$ or 4)
- Subst $x = 1$ and (one of) their y -value(s) into given $\frac{dy}{dx}$ M1 ($\frac{dy}{dx} = 7$ or 0)
- Find eqn of tgt, with their $\frac{dy}{dx}$, going through $(1, \text{their } y)$ *M1 using (one of) y value(s)
- Produce either $y = 7x - 4$ or $y = 4$ A1
- Solve simultaneously their two equations dep*M1 provided they have two
- Produce $x = \frac{8}{7}$ A1 6

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Q5, (Jun 2010, Q5)

- $\frac{d}{dx}(xy) = x \frac{dy}{dx} + y$ s.o.i. B1 Implied by e.g., $4x \frac{dy}{dx} + y$
- $\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$ B1
- Diff eqn(=0 can be implied)(solve for $\frac{dy}{dx}$ and) put $\frac{dy}{dx} = 0$ M1
- Produce only $2x + 4y = 0$ (though AEF acceptable) *A1 without any error seen
- Eliminate x or y from curve eqn & eqn(s) just produced M1
- Produce either $x^2 = 36$ or $y^2 = 9$ dep* A1 Disregard other solutions
- $(\pm 6, \mp 3)$ AEF, as the only answer ISW dep* A1 Sign aspect must be clear

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Q6, (Jan 2013, Q3)

For attempt at product rule on xy^2

$$\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x - y^2}{2xy} \text{ or } \frac{1 - x^{-2}}{2y}$$

Stationary point \rightarrow (their) $\frac{dy}{dx} = 0$ soi

$$x^2 = 1 \text{ or } y^2 = 2 \text{ or } y^4 = 4$$

$$(1, \sqrt{2}), (1, -\sqrt{2})$$

M1

B1

A1

M1

A1

A1,A1

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or changing equation to $y^2 = x + x^{-1}$
soi in the differentiating process

Award B1 for $(\pm)\frac{1}{2}(x + x^{-1})^{-\frac{1}{2}}(1 - x^{-2})$

Ignore any other values

Accept 1.41 or $4^{\frac{1}{4}}$ for $\sqrt{2}$

LHS is $k(x + y)(1 + \frac{dy}{dx})$

$k = 2$

$2y \frac{dy}{dx}$ on RHS from differentiating y^2

$y^2 + Kxy \frac{dy}{dx}$ on RHS

obtains a value of y from eg $(1 + y)^2 = 1 \times y^2$
oe

substitution of $x = 1$ and their y dependent
on at least two correct terms seen following
differentiation, even if follows subsequent
incorrect manipulation

$\frac{dy}{dx} = -\frac{3}{8}$ oe cao

M1

or $2x + 2y \frac{dy}{dx} + ky + kx \frac{dy}{dx}$

k is any positive integer

A1

B1

M1

K is any positive integer

M1

allow even if follows incorrect manipulation

M1

may be implied by $1 + \frac{dy}{dx} = \frac{1}{4} - \frac{dy}{dx}$

A1

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some terms may appear on RHS with
signs reversed

if **M0** in middle scheme, **SC1** for
three terms out of four completely
correct with $k = 2$

may appear on LHS with sign
reversed

NB $K = 2$;
may appear on LHS with signs
reversed

NB $y = -0.5$

or $\frac{dy}{dx} = \frac{2 - 1 - 0.25}{-1 - 2 + 1}$

NB $\frac{dy}{dx} = \frac{2x + 2y - y^2}{2xy - 2x - 2y}$

-0.375

Q8, (Jun 2016, Q3)

$$2y \frac{dy}{dx}$$

$$\sin 2x \frac{dy}{dx} + 2y \cos 2x$$

$$\sin 2x \frac{dy}{dx} + 2y \cos 2x - \frac{1}{x^2} + 2y \frac{dy}{dx} = 0$$

$$(\sin 2x + 2y) \frac{dy}{dx} = \frac{1}{x^2} - 2y \cos 2x \text{ oe}$$

$$\left[\frac{dy}{dx} = \right] \frac{1 - 2x^2 y \cos 2x}{(\sin 2x + 2y)x^2} \text{ oe isw}$$

B1

from differentiation of y^2

M1

correct use of Product Rule

allow sign error or one incorrect coefficient

A1

M1

collection of like terms on separate sides, need not be factorised

must be two terms in $\frac{dy}{dx}$

A1

eg $\left[\frac{dy}{dx} = \right] \frac{x^{-2} - 2y \cos 2x}{(\sin 2x + 2y)}$

A0 for eg $y = \dots$

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Q9, (Jun 2014, Q6)

| | | | |
|---|------------|---|--|
| $3y^2 \frac{dy}{dx}$ | B1 | or $2x \frac{dx}{dy}$ | if B0B0 M0 |
| $2x - 12 \frac{dy}{dx} - 8$ | B1 | $3y^2 - 8 \frac{dx}{dy} - 12$ | SC2 for $\frac{dy}{dx} =$ |
| their $3y^2 \frac{dy}{dx} - 12 \frac{dy}{dx} = 8 - 2x$ soi | M1 | their $2x \frac{dx}{dy} - 8 \frac{dx}{dy} = -3y^2 + 12$ | $\frac{1}{3} (-x^2 + 8x + 12y + 4)^{\frac{-2}{3}} \times (-2x + 8 + 12 \frac{dy}{dx})$ |
| must be two terms on each side and must follow from RHS = 0 | | must be two terms on each side must follow from RHS = 0 | M1 may be earned for setting correct denominator equal to 0 |
| $\frac{dy}{dx} = \frac{8 - 2x}{3y^2 - 12}$ oe | A1 | This mark may be implied if $\frac{dx}{dy} = 0$ is substituted and there is no evidence for an incorrect expression for $\frac{dx}{dy}$ | |
| their $3y^2 - 12 = 0$ | M1* | | $x \neq 4$ not required |
| $y = (\pm) 2$ | A1 | A0 if $\frac{dy}{dx}$ incorrect | |
| substitution of their positive y value in original equation | M1dep* | | ignore substitution of - 2 |
| $x = 10, x = -2$ and no others cao | A1 | A0 if $\frac{dy}{dx}$ incorrect | condone omission of formal statement of coordinates (10, 2) and (-2, 2) |
| | [8] | | |