Differential Equations Exam Questions MS (From OCR 4724)

Q1, (Jan 2006, Q8)

(i)	$\int(y-3)\,dy = \int(2-x)\,dx$ or equiv	M1		For separation & integration of both sides
	$\frac{1}{2}y^2 - 3y = 2x - \frac{1}{2}x^2$	A1		or $\frac{1}{2}(y-3)^2 = -\frac{1}{2}(x-2)^2$
	For an arbitrary const on one/both sides	*B1		} (or + M2 for equiv statement using limits)
	Substituting $(x, y) = (5,4)$ or (4,5) & finding 'c'	dep*M1		}
	$\frac{1}{2}y^2 - 3y = -\frac{1}{2}x^2 + 2x - \frac{3}{2}$ AEF ISW	A1	**5**	or $\frac{1}{2}(y-3)^2 = -\frac{1}{2}(x-2)^2 + 5$ AEF

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(ii)	Attempt to clear fracts (if nec) & compl square	M1		
	$a = 2, b = 3, k = 10$	A2	**3**	For all 3; SR: A1 for 1 or 2 correct

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(iii)	Circle clearly indicated in a sketch	B1		
	Centre $(2,3)$ or their (a, b)	B1√		
	Radius $\sqrt{10}$ or their $\sqrt{k}$	B1√	**3**	√ provided $k > 0$

Q2, (Jan 2007, Q9)

(i) Separate variables as $\int \sec^2 y\,dy = 2\int \cos^2 2x\,dx$	M1		seen or implied
LHS = tan y	A1		
RHS; attempt to change to double angle	M1		
Correctly shown as 1 + cos $4x$	A1		
$\int \cos 4x\,dx = \frac{1}{4}\sin 4x$	A1		
Completely correct equation (other than +c)	A1		$\tan y = x + \frac{1}{4}\sin 4x$
+c on either side	A1	**7**	not on both sides unless c_1 and c_2
(ii) Use boundary condition	M1		provided a sensible outcome would ensue
c (on RHS) = 1	A1		or $c_2 - c_1 = 1$; not fortuitously obtained
Substitute $x = \frac{1}{6}\pi$ into their eqn, produce $y = 1.05$	A1	**3**	or 4.19 or 7.33 etc. Radians only

Q3, (Jun 2007, Q8)

(i) Sep variables eg $\int \frac{1}{6-h}(dh) = \int \frac{1}{20}(dt)$	*M1		s.o.i. Or $\frac{dt}{dh} = \frac{20}{6-h}$ $\rightarrow$ M1
LHS = $-\ln(6-h)$	A1		& then $t = -20\ln(6-h)$ (+c) $\rightarrow$ A1+A1
RHS = $\frac{1}{20}t$ (+c)	A1		
Subst $t = 0, h = 1$ into equation containing 'c'	dep*M1		
Correct value of their c = $-(20)\ln 5$ **WWW**	A1		or $(20)\ln 5$ if on LHS
Produce $t = 20\ln\frac{5}{6-h}$ **WWW AG**	A1	6	Must see $\ln 5 - \ln(6-h)$
(ii) When h = 2, t = $20\ln\frac{5}{4}$ =4.46(2871)	B1	1	Accept 4.5, $4\frac{1}{2}$
(iii) Solve $10 = 20\ln\frac{5}{6-h}$ to $\frac{5}{6-h} = e^{0.5}$	M1		or $\frac{6-h}{5} = e^{-0.5}$ or suitable $\frac{1}{2}$-way stage
h = 2.97(2.9673467…)	A1	2	$6 - 5e^{-0.5}$ or $6 - e^{1.109}$
[In (ii),(iii) accept non-decimal (exact) answers but -1 once.] Accept truncated values in (ii),(iii).			
(iv) Any indication of (approximately) 6 (m)	B1	1	
			10

Q4, (Jan 2008, Q8)

(i) $\frac{dx}{dt}$ or $-kx^{\frac{1}{2}}$ or $kx^{\frac{1}{2}}$ seen	M1		k non-numerical; i.e. 1 side correct
$\frac{dx}{dt} = -kx^{\frac{1}{2}}$ or $\frac{dx}{dt} = kx^{\frac{1}{2}}$	A1	2	i.e. both sides correct
(ii) Separate variables or invert, + attempt to integrate	*M1		Based <u>only</u> on above eqns or $\frac{dx}{dt} = x^{\frac{1}{2}}, -x^{\frac{1}{2}}$
Correct result for their equation after integration	A1		Other than omission of 'c'
Subst $(t, x) = (0,2)$ into eqn containing k &/or c	dep*M1		or substitute (5,1)
Subst $(t, x) = (5,1)$ into eqn containing k <u>&</u> c	dep*M1		or substitute (0,2)
Subst $x = 0.5$ into eqn with their k & c subst	dep*M1		
$t = 8.5(8.5355339)$	A1	6	[1 d.p. requested in question]

Q5, (Jan 2009, Q9)

(i)	$\frac{d\theta}{dt} = \ldots$	B1	
	$k(160-\theta)$	B1 **(2)**	The 2 @ 'B1' are indep
(ii)	Separate variables with $(160-\theta)$ in denom; or invert	*M1	$\int \frac{1}{160-\theta} d\theta = \int k, \frac{1}{k}, 1\, dt$
	Indication that LHS = $\ln f(\theta)$	A1	If wrong ln, final 3@A = 0
	RHS = kt or $\frac{1}{k}t$ or t $(+c)$	A1	
	Subst. $t=0, \theta=20$ into equation containing 'c'	dep* M1	
	Subst $t=5, \theta=65$ into equation containing 'c' & 'k'	dep* M1	
	$c = -\ln 140$ (-4.94) ISW	A1	
	$k = \frac{1}{5}\ln\frac{140}{95}$ ($\approx$ 0.077 or 0.078) ISW	A1	
	Using their 'c' & 'k', subst t = 10 & evaluate θ	dep*M1	
	$\theta = 96(95.535714)$ $\left(95\frac{15}{28}\right)$	A1 **(9)**	
		11	

Q6, (Jan 2011, Q9)

(i)	Attempt to sep variables in the form $\int \frac{p}{(x-8)^{1/3}} dx = \int q\, dt$	M1	Or invert as $\frac{dt}{dx} = \frac{r}{(x-8)^{1/3}}$; p, q, r consts
	$\int \frac{1}{(x-8)^{1/3}} dx = k(x-8)^{2/3}$	A1	k const
	All correct $(+c)$	A1	
	For equation containing 'c'; substitute $t=0, x=72$	M1	M2 for $\int_{72}^{35} = \int_0^t$ or $\int_{35}^{72} = \int_0^t$
	Correct corresponding value of c from correct eqn	A1	
	Subst their c & $x = 35$ back into eqn	M1	
	$t = \frac{21}{8}$ or 2.63 / 2.625 [C.A.O]	A1 **7**	A2: $t = \frac{21}{8}$ or 2.63 / 2.625 WWW
(ii)	State/imply in some way that $x = 8$ when flow stops	B1	
	Substitute $x = 8$ back into equation containing numeric 'c'	M1	
	$t = 6$	A1 **3**	
		10	

Q7, (Jun 2014, Q10)

(i)	$\frac{dV}{dt} = \pm 0.01$	B1		
	by similar triangles, $\frac{h}{4.5} = \frac{r}{3}$	B1	may be implied by $r = \frac{2h}{3}$ oe	
	$\frac{dV}{dh} = \frac{4}{9}\pi h^2$ oe	B1		
	$\frac{dh}{dt} = \pm 0.01 \times$ their $\frac{dh}{dV}$ oe	M1	use of Chain rule	may follow from incorrect differentiation: expressions must be a function of either r or h or both
	$-0.01 = (\frac{4}{9}\pi h^2) \times \frac{dh}{dt}$	A1 [5]	completion to given result www	$h^2 \frac{dh}{dt} = \frac{-0.09}{4\pi} = \frac{-9}{400\pi}$
(ii)	$\int h^2 dh = \int \frac{-9}{400\pi} dt$ oe soi	M1	separation of variables	if no subsequent work, integral signs needed, but allow omission of dh or dt, but must be correctly placed if present;
	$\frac{h^3}{3} = \frac{-9}{400\pi} t (+c)$	A1		
	substitution of $t = 0$ and $h = 4.5$ in their expression following integration	M1	expression must include c and powers must be correct on each side	
	$h = \sqrt[3]{\frac{729}{8} - \frac{27t}{400\pi}}$ oe isw	A1 [4]	allow – 0.0215 or – 0.02148591…r.o.t to 4 sf or more and similarly 91.125	$91.125 = {}^{729}/_{8}$
(iii)	set $h = 0$ and solve to obtain positive t	**M1**	or $(t =) \frac{1}{3}\pi \times 3^2 \times 4.5 \div 0.01$ $(= 1350\pi)$	NB $1350\pi = 4241.150082…$
	71 minutes cao	**A1** [2]		

Q8, (Jun 2015, Q8)

(i)	$\frac{dP}{dt} = \frac{k}{P}$	**B1**	or $\frac{dP}{dt} = \frac{1}{kP}$	k should be unspecified at this stage
	$k = 100$ from $\frac{dP}{dt} = \frac{k}{P}$	**B1**	or $k = 0.01$ from $\frac{dP}{dt} = \frac{1}{kP}$	may be seen later
	$\int P\,dP = \int(\text{their } k)\,dt$	**M1***	allow $k = 1$	allow omission of $\int$ and recovery of omission of one operator for **M1*A1**
	$\frac{P^2}{2} = kt + c$	**A1**	or $t = \frac{P^2}{2k} + d$	if **M0**, **SC2** for $\ln P = kt + c$ thereafter only **M1** may be earned
	substitution of $t = 0$ and $P = 100$	**M1dep***	may follow incorrect algebraic manipulation, but equation must include c (or d)	NB $c = 5000$ or $d = -50$
	$P = \sqrt{10000 + 200t}$ or $10\sqrt{100 + 2t}$ or $P = \sqrt{200(50 + t)}$ isw cao	**A1** **[6]**		allow recovery from eg use of x for P throughout, but withhold final **A1** for eg $x = \sqrt{10000 + 200t}$
(ii)	$t = 8$, $P = 107.7$ or 108 so model was a good fit in 2008 oe	**B1**	or $t = 8.3(2)$ when $P = 108$ + comment	value of P or t must be found and correct comment made in each case; comments may be in same sentence.
	$t = 13$, $P = 112(.2)$, so model was not appropriate in 2013 oe	**B1** **[2]**	or $t = 31.9(2)$ or 32 when $P = 128$ + comment comments may be in same sentence, but both values must be referenced	if **B0B0**, **SC1** for both values found no FT marks available comments on trends, extrapolation etc do not score just ticks / crosses etc do not score

Q9, (Jun 2016, Q10)

(i)	$\frac{A}{(x+4)}+\frac{B}{(x+1)}+\frac{C}{(x+1)^2}$	**B1**		may be awarded later
	$[16+5x-2x^2]=A(x+1)^2+B(x+1)(x+4)+C(x+4)$	**M1**		allow sign errors only
	$A=-4$	**A1**	**NB** $36=-9A$	if **B0M0,** allow **SC3** for $\frac{2x+5}{(x+1)^2}-\frac{4}{x+4}$
	$C=3$	**A1**	$9=3C$	
	$B=2$ isw	**A1**	$-2=A+B$, $5=2A+5B+C$ $16=A+4B+4C$	
		[5]	**NB** $\frac{-4}{(x+4)}+\frac{2}{(x+1)}+\frac{3}{(x+1)^2}$	

(ii)	$\int \frac{dy}{y} = \int \frac{16+5x-2x^2}{(x+1)^2(x+4)}dx$	**B1**	separation of variables	allow omission of integral signs; allow omission of dy or dx but not both
	$\frac{3}{(x+1)^2} + \frac{2}{(x+1)} - \frac{4}{(x+4)}$ seen in RHS, may be embedded	**M1***	**FT** their partial fractions if two or three terms; ignore LHS	may be implied by correct integration of two of their terms
	$\frac{-3}{x+1} + 2\ln(x+1) - 4\ln(x+4) + c$	**A1FT**	**FT** their non-zero 3, 2 and 4; allow recovery from $x + 1^2$ in denominator; if brackets in log terms omitted, allow **A1** if recovery seen in substitution	allow omission of + c here
	$\ln(\frac{1}{256}) = -3 + 2\ln 1 - 4\ln 4 + c$	**M1*dep**	substitution of $x = 0$ and $y = \frac{1}{256}$; allow if error in manipulation following integration;	+ c must be included and LHS must be correctly obtained
	$c = 3$ cao	**A1**	or $A = e^{-3}$ from $y = Ae^{\frac{-3}{x+1}}\frac{(x+1)^2}{(x+4)^4}$	
	$\ln y = \frac{-3}{2+1} + 2\ln(2+1) - 4\ln(2+4) + 3$	**M1*dep**	substitution of $x = 2$; dependent on award of previous **M1M1** and numerical value found for c	allow **M1** if substitution follows incorrect manipulation eg to find expression for y
	$y = \frac{e^2}{144}$ oe	**A1** **[7]**		

Q10, (Jun 2017, Q7)

i	$\ln A - \ln(250 - A) = kt\ (+\ c)$	**M1***	allow sign error	
	valid substitution of $t = 0$ and $A = 10$ to find c	**M1dep***	**NB** $\ln 10 - \ln 240 = 0 + c$	
	$c = -\ln 24$ oe	**A1**	allow to 3 sf or more	-3.17805383
	constructive log step	**A1**	eg $\ln\left(\dfrac{A}{250 - A}\right) = kt - \ln 24$ oe	or $\ln\left(\dfrac{A}{250 - A}\right) = kt - 3.178$
	taking exponentials correctly of both sides; FT their rearrangement and/or *their* numerical c	**M1dep***	eg $\left(\dfrac{24A}{250 - A}\right) = e^{kt}$	or $\left(\dfrac{A}{250 - A}\right) = e^{kt - 3.178}$
	$[A] = \dfrac{250e^{kt}}{24 + e^{kt}}$ oe	**A1**		
	Alternatively			
	$\ln A - \ln(250 - A) = kt\ (+\ c)$	**M1***	allow sign error	
	constructive log step, may be awarded after taking exponentials	**A1**	eg $\ln\left(\dfrac{A}{250 - A}\right) = kt + c$	
	taking exponentials correctly of both sides; FT their rearrangement	**M1dep***	eg $\dfrac{A}{250 - A} = e^{kt + c}$	
	valid substitution of $t = 0$ and $A = 10$ to find c	**M1dep***	eg $\dfrac{10}{250 - 10} = e^{0 + c}$	
	$\dfrac{A}{250 - A} = e^{kt - \ln 24}$ oe	**A1**		
	$[A] = \dfrac{250e^{kt}}{24 + e^{kt}}$ oe	**A1**		
		[6]		

ii		$k = 0.05$	**B1** **[1]**		
iii		$A = 250$ [m^2]	**B1** **[1]**	ignore commentary	

Q11, (Jun 2018, Q9)

(i)	$\frac{dV}{dt} = k e^{-t}$ soi	**B1**	**B0** if x used as constant instead of k; **M** marks are still available	
	substitution of $\frac{dV}{dt} = 10$, $t = 2$	**M1**	**NB** $10 = k e^{-2}$ not from V	***alternatively, from connected rates of change***
	$V = -k e^{-t} + c$ oe	**M1***	from integration; allow omission of $+c$ here; allow eg $V = l e^{-t} + c$	$(x+1)^3 = -ke^{-t} + c$ from $\int ke^{-t} dt = \int 3(x+1)^2 dx$
	substitution of $t = 0$, $V = 0$ in their $V = -k e^{-t} + c$	**M1dep***	must see $+ c$ here; may be awarded before k found	substitution of $t = 0$, $x = 0$ (to obtain $c = 1 + k$)
	$V = 10e^2(1 - e^{-t})$ **AG**	**A1**	correct completion to given result; **A0** if 73.89… used in working	
	$x = 3$ gives $V = 63$ soi	**B1**		
	t obtained from $63 = 10e^2(1 - e^{-t})$	**M1**	**NB** 1.914688…unsupported to 3 or more sf implies **B1M1A1**	
	$t = 1.9$ to 1.915	**A1**		
		[8]		
(ii)	as t becomes large, e^{-t} becomes very small soi	**M1**	need not be explicitly stated	may be implied by 73.890….to 3 or more sf
	x obtained from $10e^2 = (x+1)^3 - 1$	**M1**		**NB** 3.215111183 unsupported to 3 or more sf justifies award of **M1M1** if working not seen
	$x = 3.22$ cao	**A1**	allow **B3** for 3.22 unsupported	
		[3]		