Area Between a Curve and the y-Axis

Q1, (OCR 4722, Jun 2008, Q5)

(i)	$\int x\,dy = \int ((y-3)^2 - 2)\,dy$	**B1**	Show $x = y^2 - 6y + 7$ convincingly
	$= \int (y^2 - 6y + 7)\,dy$ **A.G.**	**B1**	State or imply that required area $= \int x\,dy$
	$3 + \sqrt{(2+2)} = 5$, $3 + \sqrt{(14+2)} = 7$	**B1**	Use $x = 2, 14$ to show new limits of $y = 5, 7$
		3	
(ii)	$\left[\frac{1}{3}y^3 - 3y^2 + 7y\right]_5^7$	**M1**	Integration attempt, with at least one term correct
	$= (^{343}/_3 - 147 + 49) - (^{125}/_3 - 75 + 35)$	**A1**	All three terms correct
	$= 16^1/_3 - 1^2/_3$	**M1**	Attempt F(7) – F(5)
	$= 14\,^2/_3$	**A1**	Obtain $14\,^2/_3$, or exact equiv
		4	

Q2, (OCR 4722, Jun 2011, Q4)

(i)	$x + 4 = (y+1)^2$ $x + 4 = y^2 + 2y + 1$ $x = y^2 + 2y - 3$ **A.G.**	**M1**		Attempt to make x the subject	Allow M1 for $x = (y \pm 1)^2 \pm 4$ only. Allow M1 if $(y+1)^2$ becomes $y^2 + 1$, but only if clearly attempting to square the entire bracket – squaring term by term is M0. Must be from correct algebra, so M0 if eg $\sqrt{(x+4)} = \sqrt{x} + \sqrt{4}$ is used.
		A1	**2**	Verify $x = y^2 + 2y - 3$	Need to see an extra step from $(y+1)^2 - 4$ to given answer ie explicit expansion of bracket. No errors seen. **SR B1** for verification, using $y = -1 + \sqrt{(y^2 + 2y - 3 + 4)}$, and confirming relationship convincingly, or for rearranging $x = \mathrm{f}(y)$ to obtain given $y = \mathrm{f}(x)$.
ii)	$\int_1^3 (y^2 + 2y - 3)\,dy = \left[\frac{1}{3}y^3 + y^2 - 3y\right]_1^3$ $= (9 + 9 - 9) - (\frac{1}{3} + 1 - 3)$ $= (9) - (-1\frac{2}{3})$ $= 10\frac{2}{3}$	**B1**		State or imply that the required area is given by $\int_1^3 (y^2 + 2y - 3)\,dy$	No further work required beyond stating this. Allow if $3x$ appears in integral. Any further consideration of other areas is B0.
		M1		Attempt integration	Increase in power of y by 1 for at least two of the three terms. Can still get M1 if the -3 disappears, or becomes $3x$. Allow M1 for integrating a function of y that is no longer the given one, eg subtracted from 3, or using their incorrect rearrangement from part (i).
		A1ft		Obtain at least two correct terms	Allow for unsimplified coefficients. Allow follow-through on any function of y as long as at least 2 terms and related to the area required. Condone $\int$, dy or + c present.
		M1		Attempt F(3) – F(1) for their integral	Must be correct order and subtraction. This is independent of first M1 so can be given for substituting into any expression other than $y^2 + 2y - 3$, including $2y + 2$. If last term is $3x$ allow M1 for using 3 and 1 throughout integral, but M0 if x value is used instead.
		A1	**5** **7**	Obtain $10\frac{2}{3}$ aef	Must be an exact equiv so $10.\dot{6}$ is fine (but $9^5/_3$ is A0). 10.7, 10.66… or $10^2/_3 + c$ are A0. Must come from correct integral, so A0 if from $3x$. Must be given as final answer, so further work eg subtracting another area is A0 rather than ISW. Answer only is 0/5, as no evidence is provided of integration. **SR** Finding the shaded area by direct integration with respect to x (ie a C3 technique) can have **5** if done correctly, **4** if non-exact decimal given as final answer but no other partial credit.

Q3, (OCR 4722, Jun 2014, Q9)

(i)	$0.5 \times 2.5 \times (1 + 2(-3 + 2\sqrt{6.5}) + 3)$	M1*	Attempt y-values at $x = 0, 2.5, 5$ only	M0 if additional y-values found, unless not used y_1 can be exact or decimal (2.1 or better) Allow M1 for using incorrect function as long as still clearly y-values that are intended to be the original function eg $-3 + 2\sqrt{x} + 4$ (from $\sqrt{(x+4)} = \sqrt{x} + \sqrt{4}$)
	$= 10.2$	M1d*	Attempt correct trapezium rule, inc $h = 2.5$	Fully correct structure reqd, including placing of y-values The 'big brackets' must be seen, or implied by later working Could be implied by stating general rule in terms of y_0 etc, as long as these have been attempted elsewhere and clearly labelled Using x-values is M0 Can give M1, even if error in evaluating y-values as long correct intention is clear
		A1	Obtain 10.2, or better	Allow answers in the range [10.24, 10.25] if >3sf A0 if exact surd value given as final answer Answer only is 0/3 Using 2 separate trapezia can get full marks Using anything other than 2 strips of width 2.5 is M0 Using the trapezium rule on result of an integration attempt is 0/3
		[3]		
(ii)	$(5 \times 3) - 10.2 = 4.8$	M1	Attempt area of rectangle – their **(i)**	As long as $0 <$ their **(i)** < 15
		A1FT	Obtain 4.8, or better	Allow for exact surd value as well Allow answers in range [4.75, 4.80] if > 2sf
		[2]		

(iii)	$x = \frac{1}{4}(y^2 + 6y - 7)$	M1	Attempt to write as $x = f(y)$	Must be correct order of operations, but allow slip with inverse operations eg + / – , and omitting to square the $\frac{1}{2}$ Allow $y^2 + 9$ from an attempt to square $y + 3$, even if $(y+3)^2$ is not seen explicitly first Allow maximum of 1 error
		A1	Obtain $x = \frac{1}{4}(y^2 + 6y - 7)$ aef	Allow A1 as soon as any correct equation seen in format $x = f(y)$, eg $x = \frac{1}{4}(y+3)^2 - 4$ or $x = \frac{1}{4}(y^2 + 6y + 9) - 4$, and isw subsequent error
	area = $\left[\frac{1}{12}y^3 + \frac{3}{4}y^2 - \frac{7}{4}y\right]_1^3$	M1*	Attempt integration of f(*y*)	Expand bracket and increase in power by 1 for at least two terms (allow if constant term disappears) Independent of rearrangement attempt so M0M1 is possible Can gain M1 if their f(*y*) has only two terms, as long as both increase in power by 1 Allow M1 for $k(y+3)^3$, any numerical k, as the integral of $(y+3)^2$ or M1 for $k(\frac{1}{2}(y+3))^3$ from $(\frac{1}{2}(y+3))^2$ oe if their power is other than 2
		A1	Obtain $\frac{1}{12}y^3 + \frac{3}{4}y^2 - \frac{7}{4}y$ aef	Or $\frac{1}{12}(y+3)^3 - 4y$ A0 if constant term becomes $-\frac{7}{4}x$ not $-\frac{7}{4}y$
		B1	State or imply limits are $y = 1, 3$	Stated, or just used as limits in definite integral Allow B1 even if limits used incorrectly (eg wrong order, or addition) Allow B1 even if constant term is $-\frac{7}{4}x$ (or their cx)

		$= \frac{15}{4} - (-\frac{11}{12})$	M1d*	Attempt correct use of limits	Correct order and subtraction Allow M1 (BOD) if y limits used in $-\frac{7}{4}x$ (or their cx), but M0 if $x = 0, 5$ used Minimum of two terms in y Only term allowed in x is their c becoming cx Allow processing errors eg $(\frac{1}{12} \text{ x } 3)^3$ for $\frac{1}{12} \text{ x } 3^3$ Answer is given so M0 if $\frac{14}{3}$ appears with no evidence of use of limits Minimum working required is $\frac{15}{4} + \frac{11}{12}$ Allow M1 if using decimals (0.92 or better for $\frac{11}{12}$) M0 if using lower limit as $y = 0$, even if $y = 3$ is also used Limits must be from attempt at y-values, so M0 if using 0 and 5
		$= \frac{14}{3}$ **AG**	A1	Obtain $\frac{14}{3}$	Must come from exact working ie fractions or recurring decimals - correct notation required so A0 for 0.9166... A0 if $-\frac{7}{4}x$ seen in solution **SR** for candidates who find the exact area by first integrating onto the x-axis: **B4** obtain area between curve and x-axis as $10\,^1/_3$ **B1** subtract from 15 to obtain $^{14}/_3$ And, if seen in the solution, **M1A1** for $x = f(y)$ as above
			[7]		