Variable Acceleration in Two Dimensions (From OCR 4761)

Q1, (Jan 2005, Q1)

The position vector, $\mathbf{r}$, of a particle of mass 4 kg at time t is given by

$$\mathbf{r} = t^2\mathbf{i} + (5t - 2t^2)\mathbf{j},$$

where $\mathbf{i}$ and $\mathbf{j}$ are the standard unit vectors, lengths are in metres and time is in seconds.

(i) Find an expression for the acceleration of the particle. [4]

The particle is subject to a force $\mathbf{F}$ and a force $12\mathbf{j}$ N.

(ii) Find $\mathbf{F}$. [3]

Q2, (Jun 2005, Q5)

The position vector of a particle at time t is given by

$$\mathbf{r} = \tfrac{1}{2}t\mathbf{i} + (t^2 - 1)\mathbf{j},$$

referred to an origin O where $\mathbf{i}$ and $\mathbf{j}$ are the standard unit vectors in the directions of the cartesian axes Ox and Oy respectively.

(i) Write down the value of t for which the x-coordinate of the position of the particle is 2. Find the y-coordinate at this time. [2]

(ii) Show that the cartesian equation of the path of the particle is $y = 4x^2 - 1$. [2]

(iii) Find the coordinates of the point where the particle is moving at 45° to both Ox and Oy. [3]

Q3, Jun 2006, Q4)

Fig. 4 shows the unit vectors $\mathbf{i}$ and $\mathbf{j}$ in the directions of the cartesian axes Ox and Oy, respectively. O is the origin of the axes and of position vectors.

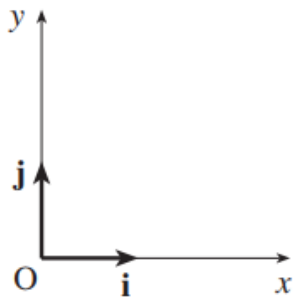

Fig. 4

The position vector of a particle is given by $\mathbf{r} = 3t\mathbf{i} + (18t^2 - 1)\mathbf{j}$ for $t \geqslant 0$, where t is time.

(i) Show that the path of the particle cuts the x-axis just once. [2]

(ii) Find an expression for the velocity of the particle at time t.

Deduce that the particle never travels in the $\mathbf{j}$ direction. [3]

(iii) Find the cartesian equation of the path of the particle, simplifying your answer. [3]

Q4, (Jan 2006, Q5)

The acceleration of a particle of mass 4 kg is given by $\mathbf{a} = (9\mathbf{i} - 4t\mathbf{j})\ \text{m s}^{-2}$, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors and t is the time in seconds.

(i) Find the acceleration of the particle when $t = 0$ and also when $t = 3$. [1]

(ii) Calculate the force acting on the particle when $t = 3$. [1]

The particle has velocity $(4\mathbf{i} + 2\mathbf{j})\ \text{m s}^{-1}$ when $t = 1$.

(iii) Find an expression for the velocity of the particle at time t. [4]

Q5, (Jan 2007, Q6)

The velocity of a model boat, $\mathbf{v}\ \text{m s}^{-1}$, is given by

$$\mathbf{v} = \begin{pmatrix} -5 \\ 10 \end{pmatrix} + t \begin{pmatrix} 6 \\ -8 \end{pmatrix},$$

where t is the time in seconds and the vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are east and north respectively.

(i) Show that when $t = 2.5$ the boat is travelling south-east (i.e. on a bearing of 135°). Calculate its speed at this time. [3]

The boat is at a point O when $t = 0$.

(ii) Calculate the bearing of the boat from O when $t = 2.5$. [4]

Q6, (Jun 2009, Q5)

The position vector of a toy boat of mass 1.5 kg is modelled as $\mathbf{r} = (2 + t)\mathbf{i} + (3t - t^2)\mathbf{j}$ where lengths are in metres, t is the time in seconds, $\mathbf{i}$ and $\mathbf{j}$ are horizontal, perpendicular unit vectors and the origin is O.

(i) Find the velocity of the boat when $t = 4$. **[3]**

(ii) Find the acceleration of the boat and the horizontal force acting on the boat. **[3]**

(iii) Find the cartesian equation of the path of the boat referred to x- and y-axes in the directions of $\mathbf{i}$ and $\mathbf{j}$, respectively, with origin O. You are not required to simplify your answer. **[2]**

Q7, (Jun 2013, Q4)

The directions of the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are east and north.

The velocity of a particle, $\mathbf{v}\ \text{m s}^{-1}$, at time t s is given by

$$\mathbf{v} = (16 - t^2)\mathbf{i} + (31 - 8t)\mathbf{j}.$$

Find the time at which the particle is travelling on a bearing of 045° and the speed of the particle at this time. **[6]**

Q8, (Jan 2011, Q4)

At time t seconds, a particle has position with respect to an origin O given by the vector

$$\mathbf{r} = \begin{pmatrix} 8t \\ 10t^2 - 2t^3 \end{pmatrix},$$

where $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are perpendicular unit vectors east and north respectively and distances are in metres.

(i) When $t = 1$, the particle is at P. Find the bearing of P from O. **[2]**

(ii) Find the velocity of the particle at time t and show that it is never zero. **[3]**

(iii) Determine the time(s), if any, when the acceleration of the particle is zero. **[3]**

Q9, (Jun 2015, Q3)

The map of a large area of open land is marked in 1 km squares and a point near the middle of the area is defined to be the origin. The vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are in the directions east and north.

At time t hours the position vectors of two hikers, Ashok and Kumar, are given by:

Ashok $\quad \mathbf{r}_A = \begin{pmatrix} -2 \\ 0 \end{pmatrix} + \begin{pmatrix} 8 \\ 1 \end{pmatrix} t,$

Kumar $\quad \mathbf{r}_K = \begin{pmatrix} 7t \\ 10 - 4t \end{pmatrix}.$

(i) Prove that the two hikers meet and give the coordinates of the point where this happens. **[4]**

(ii) Compare the speeds of the two hikers. **[3]**

Q10, (Jun 2016, Q4)

A particle is initially at the origin, moving with velocity **u**. Its acceleration **a** is constant.

At time t its displacement from the origin is $\mathbf{r} = \begin{pmatrix} x \\ y \end{pmatrix}$, where $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} t - \begin{pmatrix} 0 \\ 4 \end{pmatrix} t^2$.

(i) Write down **u** and **a** as column vectors. **[2]**

(ii) Find the speed of the particle when $t = 2$. **[3]**

(iii) Show that the equation of the path of the particle is $y = 3x - x^2$. **[3]**
