Forces and Motion in 3 Dimensions (From OCR 4761)

Q1, (Jan 2005, Q3)

(i)	$\begin{pmatrix} x \\ -7 \\ z \end{pmatrix} + \begin{pmatrix} 4 \\ y \\ -5 \end{pmatrix} + \begin{pmatrix} 5 \\ 4 \\ -7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ Equating components gives $x = -9,\ y = 3,\ z = 12$	M1 A1 A1 A1	[Allow SC 2/4 if 9, -3, -12 obtained]	4
(ii)	We need $\sqrt{5^2 + 4^2 + (-7)^2}$ $= \sqrt{90}$ or 9.48683… so 9.49 (3 s. f.)	M1 A1	Any reasonable accuracy	2
	total	6		

Q2, (Jan 2008, Q4)

(i)	Resultant is $\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} -6 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix}$ Magnitude is $\sqrt{(-2)^2 + 3^2 + 6^2} = \sqrt{49} = 7$ N	M1 A1 M1 F1	Adding the vectors. Condone spurious notation. Vector must be in proper form (penalise only once in the paper). Accept clear components. Pythagoras on **their** 3 component vector. Allow e.g. -2^2 for $(-2)^2$ even if evaluated as -4. FT **their** resultant.	4
(ii)	**F** + 2**G** + **H** = **0** So **H** = - 2**G** - **F** = $-\begin{pmatrix} -12 \\ 4 \\ 8 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ $= \begin{pmatrix} 8 \\ -5 \\ -10 \end{pmatrix}$	M1 A1 A1	Either **F** + 2**G** + **H** = **0** or **F** + 2**G** = **H** Must see attempt at **H** = – 2 **G** – **F** cao. Vector must be in proper form (penalise only once in the paper).	3
		7		

Q3, (Jun 2010, Q3)

(i)	$\begin{pmatrix}-1\\14\\-8\end{pmatrix}+\begin{pmatrix}3\\-9\\10\end{pmatrix}+\mathbf{F}=4\begin{pmatrix}-1\\2\\4\end{pmatrix}$	M1	N2L. Allow sign errors in applying N2L. Do not condone $\mathbf{F}=mg\mathbf{a}$. Allow one given force omitted.
		M1	Attempt to add $\begin{pmatrix}-1\\14\\-8\end{pmatrix}$ and $\begin{pmatrix}3\\-9\\10\end{pmatrix}$
	$\mathbf{F}=\begin{pmatrix}-6\\3\\14\end{pmatrix}$	A1 A1	Two components correct cao
		4	
(ii)	$\mathbf{v}=\begin{pmatrix}-3\\3\\6\end{pmatrix}+3\begin{pmatrix}-1\\2\\4\end{pmatrix}=\begin{pmatrix}-6\\9\\18\end{pmatrix}$ so $\begin{pmatrix}-6\\9\\18\end{pmatrix}$ m s^{-1}.	M1	$\mathbf{v}=\mathbf{u}+t\mathbf{a}$ with given **u** and **a**. Could go via **s**. If integration used, require arbitrary constant (need not be evaluated)
		A1	cao isw
	speed is $\sqrt{(-6)^2+9^2+18^2}=21$ m s^{-1}.	M1	Allow -6^2 even if interpreted as – 36. Only FT **their v**.
		F1	FT their **v** only. [Award M1 F1 for 21 seen WWW]
		4	
		8	

Q4, (Jun 2011, Q3)

(i)	$-6 = -2 \times 3$ so $y = 3 \times 3 = 9$ and $z = -4 \times 3 = -12$	M1 A1 2	May be implied Both correct [Award 2 for both correct answers seen WW]
(ii)	$\begin{pmatrix} -2 \\ 3 \\ -4 \end{pmatrix} + \begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix} = 5\mathbf{a}$	M1 B1	Use of Newton's 2nd Law in vector form for all 3 cpts of attempted resultant Treat use of wrong vectors as MR. Correct LHS
	$\mathbf{a} = \begin{pmatrix} 0.2 \\ -0.4 \\ -1 \end{pmatrix}$ so accn is $\begin{pmatrix} 0.2 \\ -0.4 \\ -1 \end{pmatrix}$ m s^{-2}	A1	The acceleration may be written as a magnitude in a given direction.
	Magnitude is $\sqrt{0.2^2 + (-0.4)^2 + (-1)^2}$ $= 1.09544\ldots$ so 1.10 m s^{-2}, (3 s. f.)	M1 F1 5	FT **their** values. Condone missing brackets. Condone no – signs. Accept 1.1. Accept surd form. Must come from a vector with 3 non-zero components for **a**
		7	

Q5, (Jun 2014, Q6)

(i)		Speed $= \sqrt{(-5)^2+0^2+(-10)^2}$	M1	For use of Pythagoras. Accept $\sqrt{5^2+10^2}$.	
		$= 11.2 \text{ m s}^{-1}$ (11.18)	A1	Accept $\sqrt{125}$ or $5\sqrt{5}$	
		$\tan\theta = \frac{5}{10}$	M1	Complete method for correct angle; may use $\sin\theta = \frac{5}{11.2}$, $\cos\theta = \frac{10}{11.2}$.	
		$\theta = 26.6°$	A1 [4]	Allow 153.4°, 206.6°	
(ii)		$\begin{pmatrix} 0 \\ 0 \\ -980 \end{pmatrix}$ her weight	B1	The descriptions should be linked to the forces, either by the layout of the answer or by suitable text. If not, assume that the forces they refer to are in the order given here (which is the same as the question).	
		$\begin{pmatrix} 0 \\ 0 \\ 880 \end{pmatrix}$ resistance to her vertical motion	B1	Accept "Air resistance", "Arms stretched out" and similar statements. Condone mention of a parachute.	
		$\begin{pmatrix} 50 \\ -20 \\ 0 \end{pmatrix}$ force from the power unit	B1 [3]		
(iii)		Resultant force $= \begin{pmatrix} 50 \\ -20 \\ -100 \end{pmatrix}$	B1	May be implied	
		Acceleration $= \begin{pmatrix} 0.5 \\ -0.2 \\ -1 \end{pmatrix}$	B1	Newton's 2nd Law	
		Magnitude $= \sqrt{0.5^2+(-0.2)^2+1^2} = 1.1357...$ So 1.14 to 3 s.f.	B1 [3]	Answer given. Allow FT from sign errors. Accept $\lvert\mathbf{F}\rvert \div 100$	

(iv)		$\mathbf{v} = \mathbf{u} + \mathbf{a}t$	M1	FT their **a** for the first 5 marks of this part. Vectors must be seen or implied. Accept valid integration.
		$\mathbf{v} = \begin{pmatrix} -5 \\ 0 \\ -10 \end{pmatrix} + \begin{pmatrix} 0.5 \\ -0.2 \\ -1 \end{pmatrix} t$	A1	
		$\mathbf{r} = \mathbf{r}_0 + \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	M1	Vectors must be seen or implied. Accept valid integration. Condone no $\mathbf{r}_0$ for this M mark
		$\mathbf{r} = \begin{pmatrix} -75 \\ 90 \\ 750 \end{pmatrix} + \begin{pmatrix} -5 \\ 0 \\ -10 \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0.5 \\ -0.2 \\ -1 \end{pmatrix} t^2$	A1	
		When $t = 30$	M1	Vectors must be seen or implied.
		$\mathbf{r} = \begin{pmatrix} -75 - 150 + 225 \\ 90 + 0 - 90 \\ 750 - 300 - 450 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, as required	E1	CAO
				SC1 to replace the first 4 marks of this section if the acceleration is taken to be **g** but the answer is otherwise correct.
			[6]	
(v)		When $t = 30$, $\mathbf{v} = \begin{pmatrix} 10 \\ -6 \\ -40 \end{pmatrix}$	M1	There must be an attempt to work out at least the vertical component of the velocity at $t = 30$. This mark is not dependent on a correct answer.
		The vertical component of the velocity is too fast for a safe landing	A1	Accept an argument based on speed derived from a vector.
			[2]	

Q6, (Jun 2013, Q3)

(i)	**p** $\sqrt{(-1)^2+(-1)^2+5^2}=\sqrt{27}$ **q** $\sqrt{(-1)^2+(-4)^2+2^2}=\sqrt{21}$ **r** $\sqrt{2^2+5^2+0^2}=\sqrt{29}$	M1	Use of Pythagoras Note Magnitudes are 5.196, 4.583 and 5.385 respectively
	Greatest magnitude: **r**	A1	
		[2]	
(ii)	Weight $=\begin{pmatrix}0\\0\\-4\end{pmatrix}$	B1	Condone $g=9.8$ giving weight is $\begin{pmatrix}0\\0\\-3.92\end{pmatrix}$ N. Accept 4↓.
	$\mathbf{p}+\mathbf{q}+\mathbf{r}+\mathbf{weight}=\begin{pmatrix}0\\0\\3\end{pmatrix}$		$g=9.8$ gives $\begin{pmatrix}0\\0\\3.08\end{pmatrix}$
	$0.4\mathbf{a}=\begin{pmatrix}0\\0\\3\end{pmatrix}$	B1	Relevant attempt at Newton's 2nd Law. The total force must be expressed as a vector in some form. For this mark allow the weight to be missing, in the wrong component or to have the wrong sign. Condone *mg* in place of *m* for this mark only.
	Magnitude of acceleration is 7.5 m s^{-2}	B1	CAO apart from using $g=9.8 \Rightarrow a=7.7$
	Direction is vertically upwards	B1	
		[4]	