Further collisions

Understand and be able to use the concept of impulse.

Analyse collisions involving oblique impacts, including the use of the coefficient of restitution. Questions involving successive impacts may be set.

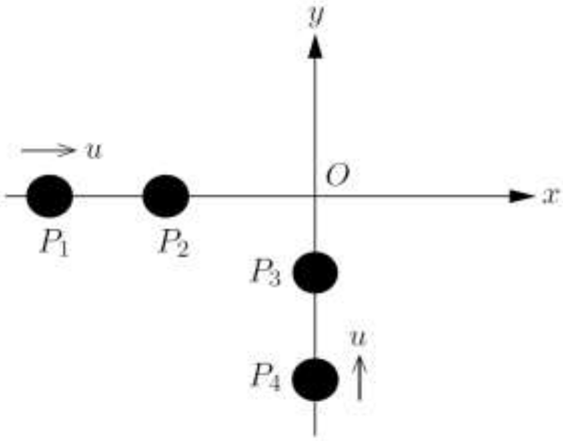
Q1, (STEP II, 2006, Q10)

Three particles, A, B and C, of masses m, km and $3m$ respectively, are initially at rest lying in a straight line on a smooth horizontal surface. Then A is projected towards B at speed u. After the collision, B collides with C. The coefficient of restitution between A and B is $\frac{1}{2}$ and the coefficient of restitution between B and C is $\frac{1}{4}$.

(i) Find the range of values of k for which A and B collide for a second time.

(ii) Given that $k = 1$ and that B and C are initially a distance d apart, show that the time that elapses between the two collisions of A and B is $\dfrac{60d}{13u}$.

Q2, (STEP II, 2009, Q10)



Four particles P_1, P_2, P_3 and P_4, of masses m_1, m_2, m_3 and m_4, respectively, are arranged on smooth horizontal axes as shown in the diagram.

Initially, P_2 and P_3 are stationary, and both P_1 and P_4 are moving towards O with speed u. Then P_1 and P_2 collide, at the same moment as P_4 and P_3 collide. Subsequently, P_2 and P_3 collide at O, as do P_1 and P_4 some time later. The coefficient of restitution between each pair of particles is e, and $e > 0$.

Show that initially P_2 and P_3 are equidistant from O.

Q3, (STEP II, 2012, Q11)

A small block of mass km is initially at rest on a smooth horizontal surface. Particles P_1, P_2, P_3, … are fired, in order, along the surface from a fixed point towards the block. The mass of the ith particle is im ($i = 1, 2, \ldots$) and the speed at which it is fired is u/i. Each particle that collides with the block is embedded in it. Show that, if the nth particle collides with the block, the speed of the block after the collision is

$$\frac{2nu}{2k + n(n+1)}.$$

In the case $2k = N(N+1)$, where N is a positive integer, determine the number of collisions that occur. Show that the total kinetic energy lost in all the collisions is

$$\tfrac{1}{2}mu^2\left(\sum_{n=2}^{N+1}\frac{1}{n}\right).$$

Q4, (STEP III, 2018, Q10)

Two identical smooth spheres P and Q can move on a smooth horizontal table. Initially, P moves with speed u and Q is at rest. Then P collides with Q. The direction of travel of P before the collision makes an acute angle α with the line joining the centres of P and Q at the moment of the collision. The coefficient of restitution between P and Q is e where $e < 1$.

As a result of the collision, P has speed v and Q has speed w, and P is deflected through an angle θ.

(i) Show that

$$u\sin\alpha = v\sin(\alpha+\theta)$$

and find an expression for w in terms of v, θ and α.

(ii) Show further that

$$\sin\theta = \cos(\theta+\alpha)\sin\alpha + e\sin(\theta+\alpha)\cos\alpha$$

and find an expression for $\tan\theta$ in terms of $\tan\alpha$ and e.

Find, in terms of e, the maximum value of $\tan\theta$ as α varies.
