STEP III - Centres of Mass

Centre of mass

Understand and be able to apply the principle that the effect of gravity is equivalent to a single force acting at the body's centre of mass.

Find the position of the centre of mass of a uniform rigid body using symmetry.

Determine the centre of mass of a system of particles or the centre of mass of a composite rigid body.

Use integration to determine the position of the centre of mass of a uniform lamina or a uniform solid of revolution.

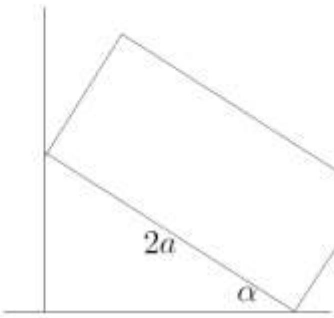
Q1, (STEP II, 1998, Q9)

A light smoothly jointed planar framework in the form of a regular hexagon $ABCDEF$ is suspended smoothly from A and a weight 1kg is suspended from C. The framework is kept rigid by three light rods BD, BE and BF. What is the direction and magnitude of the supporting force which must be exerted on the framework at A?

Indicate on a labelled diagram which rods are in thrust (compression) and which are in tension.

Find the magnitude of the force in BE.

Q2, (STEP I, 2010, Q9)



The diagram shows a uniform rectangular lamina with sides of lengths $2a$ and $2b$ leaning against a rough vertical wall, with one corner resting on a rough horizontal plane. The plane of the lamina is vertical and perpendicular to the wall, and one edge makes an angle of α with the horizontal plane. Show that the centre of mass of the lamina is a distance $a\cos\alpha + b\sin\alpha$ from the wall.

The coefficients of friction at the two points of contact are each μ and the friction is limiting at both contacts. Show that

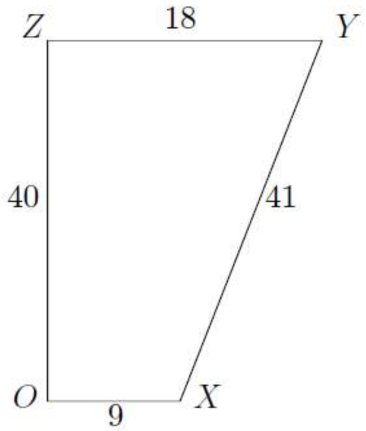
$$a\cos(2\lambda + \alpha) = b\sin\alpha\,,$$

where $\tan\lambda = \mu$.

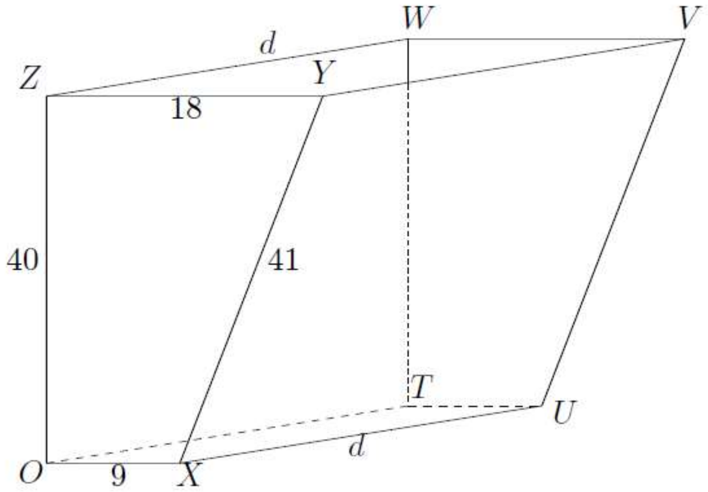
Show also that if the lamina is square, then $\lambda = \frac{\pi}{4} - \alpha$.

Q3, (STEP II, 2009, Q9)

(i) A uniform lamina $OXYZ$ is in the shape of the trapezium shown in the diagram. It is right-angled at O and Z, and OX is parallel to YZ. The lengths of the sides are given by $OX = 9\,\text{cm}$, $XY = 41\,\text{cm}$, $YZ = 18\,\text{cm}$ and $ZO = 40\,\text{cm}$. Show that its centre of mass is a distance $7\,\text{cm}$ from the edge OZ.



(ii) The diagram shows a tank with no lid made of thin sheet metal. The base $OXUT$, the back $OTWZ$ and the front $XUVY$ are rectangular, and each end is a trapezium as in part (i) . The width of the tank is $d\,\text{cm}$.



Show that the centre of mass of the tank, when empty, is a distance

$$\frac{3(140 + 11d)}{5(12 + d)}\ \text{cm}$$

from the back of the tank.

The tank is then filled with a liquid. The mass per unit volume of this liquid is k times the mass per unit area of the sheet metal. In the case $d = 20$, find an expression for the distance of the centre of mass of the filled tank from the back of the tank.

Q4, (STEP I, 2011, Q11)

A thin non-uniform bar AB of length $7d$ has centre of mass at a point G, where $AG = 3d$. A light inextensible string has one end attached to A and the other end attached to B. The string is hung over a smooth peg P and the bar hangs freely in equilibrium with B lower than A. Show that

$$3\sin\alpha = 4\sin\beta\,,$$

where α and β are the angles PAB and PBA, respectively.

Given that $\cos\beta = \frac{4}{5}$ and that α is acute, find in terms of d the length of the string and show that the angle of inclination of the bar to the horizontal is $\arctan\frac{1}{7}$.