Energy, work, and power

Understand and use the concepts of energy (kinetic and potential), work, and power.

Understand and use the principle of conservation of energy.

Q1, (STEP II, 2017, Q10)

A car of mass m makes a journey of distance $2d$ in a straight line. It experiences air resistance and rolling resistance so that the total resistance to motion when it is moving with speed v is $Av^2 + R$, where A and R are constants.

The car starts from rest and moves with constant acceleration a for a distance d. Show that the work done by the engine for this half of the journey is

$$\int_0^d (ma + R + Av^2)\,\mathrm{d}x$$

and that it can be written in the form

$$\int_0^w \frac{(ma + R + Av^2)v}{a}\,\mathrm{d}v\,,$$

where $w = \sqrt{2ad}$.

For the second half of the journey, the acceleration of the car is $-a$.

(i) In the case $R > ma$, show that the work done by the engine for the whole journey is

$$2Aad^2 + 2Rd\,.$$

(ii) In the case $ma - 2Aad < R < ma$, show that at a certain speed the driving force required to maintain the constant acceleration falls to zero.

Thereafter, the engine does no work (and the driver applies the brakes to maintain the constant acceleration). Show that the work done by the engine for the whole journey is

$$2Aad^2 + 2Rd + \frac{(ma - R)^2}{4Aa}\,.$$

Q2, (STEP II, 2004, Q11)

The maximum power that can be developed by the engine of train A, of mass m, when travelling at speed v is $Pv^{3/2}$, where P is a constant. The maximum power that can be developed by the engine of train B, of mass $2m$, when travelling at speed v is $2Pv^{3/2}$. For both A and B resistance to motion is equal to kv, where k is a constant.

For $t \leqslant 0$, the engines are crawling along at very low equal speeds. At $t = 0$, both drivers switch on full power and at time t the speeds of A and B are v_A and v_B, respectively.

(i) Show that

$$v_A = \frac{P^2\left(1 - e^{-kt/2m}\right)^2}{k^2}$$

and write down the corresponding result for v_B.

(ii) Find v_A and v_B when $9v_A = 4v_B$. [You may find the substitution $v_A = u^2$ useful.]

(iii) Both engines are switched off when $9v_A = 4v_B$. Show that thereafter $k^2v_B^2 = 4P^2v_A$.

Q3, (STEP II, 2009, Q11)

A train consists of an engine and n trucks. It is travelling along a straight horizontal section of track. The mass of the engine and of each truck is M. The resistance to motion of the engine and of each truck is R, which is constant. The maximum power at which the engine can work is P.

Obtain an expression for the acceleration of the train when its speed is v and the engine is working at maximum power.

The train starts from rest with the engine working at maximum power. Obtain an expression for the time T taken to reach a given speed V, and show that this speed is only achievable if

$$P > (n+1)RV\,.$$

(i) In the case when $(n+1)RV/P$ is small, use the approximation $\ln(1-x) \approx -x - \frac{1}{2}x^2$ (valid for small x) to obtain the approximation

$$PT \approx \tfrac{1}{2}(n+1)MV^2$$

and interpret this result.

(ii) In the general case, the distance moved from rest in time T is X. *Write down*, with explanation, an equation relating P, T, X, M, V, R and n and hence show that

$$X = \frac{2PT - (n+1)MV^2}{2(n+1)R}\,.$$
