STEP II - Collisions

Collisions

Understand the mechanics of collisions in simple situations.

Understand and use the principle of conservation of momentum and, when appropriate, the conservation of energy applied to collisions.

Understand and use the coefficient of restitution (e) for collisions, including the special cases $e = 1$ and $e = 0$.

Questions involving successive impacts may be set.

Knowledge of oblique impacts will not be required.

Q1, (STEP II, 2006, Q10)

Three particles, A, B and C, of masses m, km and $3m$ respectively, are initially at rest lying in a straight line on a smooth horizontal surface. Then A is projected towards B at speed u. After the collision, B collides with C. The coefficient of restitution between A and B is $\frac{1}{2}$ and the coefficient of restitution between B and C is $\frac{1}{4}$.

(i) Find the range of values of k for which A and B collide for a second time.

(ii) Given that $k = 1$ and that B and C are initially a distance d apart, show that the time that elapses between the two collisions of A and B is $\dfrac{60d}{13u}$.

Q2, (STEP II, 2008, Q10)

The lengths of the sides of a rectangular billiards table $ABCD$ are given by $AB = DC = a$ and $AD = BC = 2b$. There are small pockets at the midpoints M and N of the sides AD and BC, respectively. The sides of the table may be taken as smooth vertical walls.

A small ball is projected along the table from the corner A. It strikes the side BC at X, then the side DC at Y and then goes directly into the pocket at M. The angles BAX, CXY and DYM are α, β and γ respectively. On each stage of its path, the ball moves with constant speed in a straight line, the speeds being u, v and w respectively. The coefficient of restitution between the ball and the sides is e, where $e > 0$.

(i) Show that $\tan\alpha \tan\beta = e$ and find γ in terms of α.

(ii) Show that $\tan\alpha = \dfrac{(1+2e)b}{(1+e)a}$ and deduce that the shot is possible whatever the value of e.

(iii) Find an expression in terms of e for the fraction of the kinetic energy of the ball that is lost during the motion.

Q3, (STEP II, 2010, Q10)

(i) In an experiment, a particle A of mass m is at rest on a smooth horizontal table. A particle B of mass bm, where $b > 1$, is projected along the table directly towards A with speed u. The collision is perfectly elastic.

Find an expression for the speed of A after the collision in terms of b and u, and show that, irrespective of the relative masses of the particles, A cannot be made to move at twice the initial speed of B.

(ii) In a second experiment, a particle B_1 is projected along the table directly towards A with speed u. This time, particles B_2, B_3, ..., B_n are at rest in order on the line between B_1 and A. The mass of B_i ($i = 1, 2, \ldots, n$) is $\lambda^{n+1-i}m$, where $\lambda > 1$. All collisions are perfectly elastic. Show that, by choosing n sufficiently large, there is no upper limit on the speed at which A can be made to move.

In the case $\lambda = 4$, determine the least value of n for which A moves at more than $20u$. You may use the approximation $\log_{10} 2 \approx 0.30103$.

Q4, (STEP II, 2011, Q9)

Two particles, A of mass $2m$ and B of mass m, are moving towards each other in a straight line on a smooth horizontal plane, with speeds $2u$ and u respectively. They collide directly. Given that the coefficient of restitution between the particles is e, where $0 < e \leqslant 1$, determine the speeds of the particles after the collision.

After the collision, B collides directly with a smooth vertical wall, rebounding and then colliding directly with A for a second time. The coefficient of restitution between B and the wall is f, where $0 < f \leqslant 1$. Show that the velocity of B after its second collision with A is

$$\tfrac{2}{3}(1-e^2)u - \tfrac{1}{3}(1-4e^2)fu$$

towards the wall and that B moves towards (not away from) the wall for all values of e and f.

Q5, (STEP I, 2014, Q10)

(i) A uniform spherical ball of mass M and radius R is released from rest with its centre a distance $H + R$ above horizontal ground. The coefficient of restitution between the ball and the ground is e. Show that, after bouncing, the centre of the ball reaches a height $R + He^2$ above the ground.

(ii) A second uniform spherical ball, of mass m and radius r, is now released from rest together with the first ball (whose centre is again a distance $H + R$ above the ground when it is released). The two balls are initially one on top of the other, with the second ball (of mass m) above the first. The two balls separate slightly during their fall, with their centres remaining in the same vertical line, so that they collide immediately after the first ball has bounced on the ground. The coefficient of restitution between the balls is also e. The centre of the second ball attains a height h above the ground.

Given that $R = 0.2$, $r = 0.05$, $H = 1.8$, $h = 4.5$ and $e = \frac{2}{3}$, determine the value of M/m.

Q6, (STEP II, 2013, Q11)

Three identical particles lie, not touching one another, in a straight line on a smooth horizontal surface. One particle is projected with speed u directly towards the other two which are at rest. The coefficient of restitution in all collisions is e, where $0 < e < 1$.

(i) Show that, after the second collision, the speeds of the particles are $\frac{1}{2}u(1-e)$, $\frac{1}{4}u(1-e^2)$ and $\frac{1}{4}u(1+e)^2$. Deduce that there will be a third collision whatever the value of e.

(ii) Show that there will be a fourth collision if and only if e is less than a particular value which you should determine.

Q7, (STEP I, 2016, Q10)

Four particles A, B, C and D are initially at rest on a smooth horizontal table. They lie equally spaced a small distance apart, in the order $ABCD$, in a straight line. Their masses are λm, m, m and m, respectively, where $\lambda > 1$.

Particles A and D are simultaneously projected, both at speed u, so that they collide with B and C (respectively). In the following collision between B and C, particle B is brought to rest. The coefficient of restitution in each collision is e.

(i) Show that $e = \dfrac{\lambda - 1}{3\lambda + 1}$ and deduce that $e < \frac{1}{3}$.

(ii) Given also that C and D move towards each other with the same speed, find the value of λ and of e.

Q8, (STEP I, 2017, Q10)

Particles P_1, P_2, ... are at rest on the x-axis, and the x-coordinate of P_n is n. The mass of P_n is $\lambda^n m$. Particle P, of mass m, is projected from the origin at speed u towards P_1. A series of collisions takes place, and the coefficient of restitution at each collision is e, where $0 < e < 1$. The speed of P_n immediately after its first collision is u_n and the speed of P_n immediately after its second collision is v_n. No external forces act on the particles.

(i) Show that $u_1 = \dfrac{1+e}{1+\lambda}u$ and find expressions for u_n and v_n in terms of e, λ, u and n.

(ii) Show that, if $e > \lambda$, then each particle (except P) is involved in exactly two collisions.

(iii) Describe what happens if $e = \lambda$ and show that, in this case, the fraction of the initial kinetic energy lost approaches e as the number of collisions increases.

(iv) Describe what happens if $\lambda e = 1$. What fraction of the initial kinetic energy is eventually lost in this case?
