STEP I – Introductory Questions

Algebra and functions

Know, understand and use the laws of indices for all rational exponents.

Use and manipulate surds, including rationalising the denominator.

Work with quadratic functions and their graphs; the discriminant of a quadratic function, including the conditions for real and repeated roots; completing the square; solution of quadratic equations including solving quadratic equations in a function of the unknown.

Solve simultaneous equations in two (***or more***) variables by elimination and by substitution; including, ***for example***, one linear and one quadratic equation.

Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation, and simple algebraic division; use of the factor theorem ***and the remainder theorem; use of equating coefficients in identities.***

Q1, (STEP I, 2004, Q1)

Skill: Making connections with previous parts of the question

(i) Express $\left(3+2\sqrt{5}\right)^3$ in the form $a+b\sqrt{5}$ where a and b are integers.

(ii) Find the positive integers c and d such that $\sqrt[3]{99-70\sqrt{2}} = c - d\sqrt{2}$.

(iii) Find the two real solutions of $x^6 - 198x^3 + 1 = 0$.

Q2, (STEP I, 2006, Q2)

Skill: Drawing accurate diagrams

A small goat is tethered by a rope to a point at ground level on a side of a square barn which stands in a large horizontal field of grass. The sides of the barn are of length $2a$ and the rope is of length $4a$. Let A be the area of the grass that the goat can graze. Prove that $A \leqslant 14\pi a^2$ and determine the minimum value of A.

Q3, (STEP I, 2009, Q2)

Skill: Using intuition to "spot" solutions.

A curve has the equation

$$y^3 = x^3 + a^3 + b^3,$$

where a and b are positive constants. Show that the tangent to the curve at the point $(-a, b)$ is

$$b^2y - a^2x = a^3 + b^3.$$

In the case $a = 1$ and $b = 2$, show that the x-coordinates of the points where the tangent meets the curve satisfy

$$7x^3 - 3x^2 - 27x - 17 = 0.$$

Hence find positive integers p, q, r and s such that

$$p^3 = q^3 + r^3 + s^3.$$

Q4, (STEP I, 2010, Q1)

Skill: Taking inspiration from previously done work to develop method

Given that

$$5x^2 + 2y^2 - 6xy + 4x - 4y \equiv a\,(x - y + 2)^2 + b\,(cx + y)^2 + d\,,$$

find the values of the constants a, b, c and d.

Solve the simultaneous equations

$$5x^2 + 2y^2 - 6xy + 4x - 4y = 9\,,$$
$$6x^2 + 3y^2 - 8xy + 8x - 8y = 14\,.$$

Q5, (STEP I, 2013, Q1)

Skill: Spotting similarities/slight differences between questions to develop methods

(i) Use the substitution $\sqrt{x} = y$ (where $y \geqslant 0$) to find the real root of the equation

$$x + 3\sqrt{x} - \tfrac{1}{2} = 0\,.$$

(ii) Find all real roots of the following equations:

(a) $x + 10\sqrt{x+2} - 22 = 0\,;$

(b) $x^2 - 4x + \sqrt{2x^2 - 8x - 3} - 9 = 0\,.$

Q6, (STEP II, 2004, Q1)

Skill: Knowing when to check for validity of solutions obtained.

Find all real values of x that satisfy:

(i) $\sqrt{3x^2 + 1} + \sqrt{x} - 2x - 1 = 0\,;$

(ii) $\sqrt{3x^2 + 1} - 2\sqrt{x} + x - 1 = 0\,;$

(iii) $\sqrt{3x^2 + 1} - 2\sqrt{x} - x + 1 = 0\,.$
