Z-Values in the Normal Distribution

\[ X \sim N(30, 16) \quad Y \sim N(20, 9) \quad Z \sim N(0, 1) \]

\[ P(X > 38) \quad P(Y > 26) \quad P(Z > 2) \]

0.0228   0.0228   0.0228

The reason these answers are all the same is that their boundaries are exactly 2 standard deviations from
the mean. No of standard deviations from the mean in
Normal Distributions is known as the Z-value.

e.g. Calculate the z-value for \( X \sim N(27, 40) \)
where \( X = 18 \).

\[
Z = \frac{X - \mu}{\sigma} = \frac{18 - 27}{\sqrt{40}} = -1.423
\]

Calculating Unknown Mean or Variance of a Normal Distribution

e.g. Find \( \mu \) for \( X \sim N(\mu, 12) \) given that \( P(X > 6) = 0.72 \)

1) Calculate z-value for given boundary algebraically

\[
Z = \frac{6 - \mu}{\sqrt{12}}
\]
2. Calculate the corresponding z-value for $Z \sim N(0, 1)$ using given probability.

3. Solve the two equations for $z$ simultaneously to find the unknowns.

\[
\frac{6 - \mu}{\sqrt{12}} = -0.58284
\]

\[
6 - \mu = -2.0190 \Rightarrow \mu = 8.0190
\]

\[
e.g. \text{ For } X \sim N(20, \sigma^2), \text{ given that } P(X > 22) = 0.3, \text{ find the standard deviation } \sigma.
\]

\[
z = \frac{22 - 20}{\sigma} = \frac{2}{\sigma}
\]

$Z \sim N(0, 1)$

\[
\Rightarrow \frac{2}{\sigma} = 0.5244 \Rightarrow \sigma = 3.8139
\]
The random variable $T$ is normally distributed with mean $\mu$ and standard deviation $\sigma$. It is given that $P(T > 80) = 0.05$ and $P(T > 50) = 0.75$. Find the values of $\mu$ and $\sigma$.

$$T \sim N(\mu, \sigma^2)$$

$$z = \frac{80 - \mu}{\sigma}$$

$$z = \frac{50 - \mu}{\sigma}$$

Inversion

<table>
<thead>
<tr>
<th>Inverse Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area : 0.95</td>
</tr>
<tr>
<td>$\sigma$ : 1</td>
</tr>
<tr>
<td>$\mu$ : 0</td>
</tr>
</tbody>
</table>

\[
x_{\text{Inv}} =
\]

\[
z = 1.644853667
\]

\[
\Rightarrow \frac{80 - \mu}{\sigma} = 1.6449
\]

\[
80 - \mu = 1.6449\sigma
\]

\[
\mu + 1.6449\sigma = 80
\]

Inversion

<table>
<thead>
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<tbody>
<tr>
<td>Area : 0.25</td>
</tr>
<tr>
<td>$\sigma$ : 1</td>
</tr>
<tr>
<td>$\mu$ : 0</td>
</tr>
</tbody>
</table>

\[
x_{\text{Inv}} =
\]

\[
z = -0.674489579
\]

\[
\Rightarrow \frac{50 - \mu}{\sigma} = -0.67449
\]

\[
50 - \mu = -0.67449\sigma
\]

\[
\mu - 0.67449\sigma = 50
\]

Can be solved using calculator functions

\[
\begin{align*}
1x + 1.6449\sigma &= 80 \\
1x - 0.67449\sigma &= 50
\end{align*}
\]

\[
x = 58.7241473
\]

\[
\sigma = 12.93443535
\]