Numerical Methods for Solving Equations

Locating Roots Via Change of Sign

e.g. Show that the equation \( x^2 + 7x - 12 = 0 \) has a root between \( x = 1 \) and \( x = 2 \).

**Theory**

Graphically, a function has a root when its graph crosses the \( x \)-axis. If it is above the axis before it crosses then it is below the \( x \)-axis after it crosses.

\[
f(x) = x^2 + 7x - 12
\]

1. Sub in the boundaries of the interval in which the root is thought to lie and compare the results to 0
   
   \[ f(1) = (1)^2 + 7(1) - 12 = -4 < 0 \]
   \[ f(2) = (2)^2 + 7(2) - 12 = 6 > 0 \]

2. Write the following conclusion:
   
   **Change of signs**: since function is continuous in the interval \([1, 2]\), a root must be present

   \( \therefore \) Root lies in interval \([1, 2]\)

**Failure of Change of Sign Method**

Here \( f(1) \) is negative
\( f(2) \) is also negative
But clearly there are two roots in the interval but no change of sign.

The method has failed.
The function has a discontinuity at the y-axis. This method only works when a function is continuous in a given interval.

\[ f(-1) < 0 \]
\[ f(1) > 0 \]
\[ \Rightarrow \text{Change of sign but no root} \]

\[ f(1) > 0 \]
\[ f(2) > 0 \]
\[ \Rightarrow \text{No change of sign} \]
\[ \text{However there is a root in the interval } [1, 2] \]
\[ \Rightarrow \text{Method has failed} \]

e.g. Prove that for \( f(x) = 3 + x^4 - x^5 \), \( x = 1.864 \) is a root correct to 3 d.p.

Here, we need to prove that anything that rounds to 1.441 is an interval containing the root. We know to 3 d.p, \( x \) must lie within the interval \([1.8635, 1.8645]\)

\[ f(1.8635) = 1.38 \times 10^{-3} > 0 \]
\[ \Rightarrow \text{Change of sign and since function is continuous in interval } [1.8635, 1.8645] \]
\[ f(1.8645) = -5.13 \times 10^{-3} < 0 \]
\[ \text{the root is 1.864 to 3 d.p.} \]