Further Polynomial Division

In Yr 1, polynomial division only occurred with a linear denominator. In Yr 2, the order of the denominator could be linear or greater.

e.g. Simplify \((x^4 + 3x^2 + 9x - 12) \div (x + 2)\)

\[
\begin{array}{c|ccccc}
& x^3 & -2x^2 & +7x & -5 \\
\hline
x & x^4 & -2x^3 & +7x^2 & -5x \\
+2 & 2x^3 & -4x^2 & +14x & -10
\end{array}
\]

\[Q = x^3 - 2x^2 + 7x - 5, \quad r = -2\]

\[\frac{7}{2} = 3 + \frac{1}{2}\]

\[\frac{x^4 - 29x^2 - 40x - 40}{x^2 - 3x - 8}\]

\[
\begin{array}{c|ccccc}
& 2x^2 & +6x & + 5 \\
\hline
x^2 & 2x^4 & +6x^3 & +5x^2 \\
-3x & -6x^3 & -18x^2 & -15x \\
-8 & -16x^2 & -48x & -40
\end{array}
\]

\[2x^2 + 6x + 5 + \frac{23x}{x^2 - 3x - 8}\]

Must add up to \(-40x\). \(r = 23x\)

Need \(-40\) in answer

\(\therefore\) no constant remainder
Express \( \frac{9x^2 + 20x - 10}{(x + 2)(3x - 1)} \) in partial fractions.

Notice that the denominator has the same order as the numerator. This means the division algorithm will give rise to a constant term

\[
(3x^2 - x + 6x - 2) = 3x^2 + 5x - 2
\]

\[
\begin{array}{c|cc|c}
& 3x^2 & 9x^2 & 3x & -4 \\
\hline
3x^2 & 9x^2 & r &= 5x - 4 \\
+5x & 15x & = & 3 + \frac{5x-4}{(x+2)(3x-1)} \\
-2 & -6 & \\
\end{array}
\]

Do division by grid method

**Key Point**: If the numerator is too-heavy i.e. too degree \( \geq \) the denominator, a division must be performed before partial fractions can be found.

Concentrating on just the partial fractions and ignoring the quotient:

\[
\frac{5x-4}{(x+2)(3x-1)} = \frac{A}{x+2} + \frac{B}{3x-1}
\]

\[
5x - 4 \equiv A(3x - 1) + B(x + 2)
\]

Let \( x = -2 \Rightarrow -14 = -7A \Rightarrow A = 2 \)

\( x = \frac{1}{3} \Rightarrow -\frac{7}{3} = \frac{7}{3}B \Rightarrow B = -1 \)
\[ 3 + \frac{2}{x+2} - \frac{1}{3x-1} \]