Composite Functions

A function is a ‘thing’ that ‘eats’ a number, does something to it then ‘spits’ out the result.

\[ f(x) = x^2 + 2 \]

Here, \( f \) is the function which ‘eats’ a number, squares it, adds 2, then spits out the result.

Sometimes we can apply multiple functions to a quantity.

\[ \begin{align*}
  f(x) &= e^x \\
  g(x) &= x^2 - 3 \\
  f(g(x)) &= f(x^2 - 3) = e^{x^2 - 3} \\
  g(f(x)) &= g(e^x) = (e^x)^2 - 3 \\
  ggf(y) &= gg(e^{y^2}) = g((e^{y^2})^2 - 3) = ((e^{y^2})^2 - 3)^2 - 3
\end{align*} \]

Numerical expressions can also be fed into a function.

\[ \begin{align*}
  fgf(2) &= f(g(e^2)) = f(e^4 - 3) = e^{e^4 - 3}
\end{align*} \]

Key point: the function closest to the bracket is the one that gets applied first.
The functions $f$ and $g$ are defined for all real values of $x$ by

$$f(x) = 3x - 2 \quad \text{and} \quad g(x) = 3x + 7.$$  

Find the exact coordinates of the point at which $y = f(x)g(x)$ meets the $x$-axis,

(i) the graph of $y = f(x)g(x)$ meets the $x$-axis,

$$y = f(x)g(x) = f(3x + 7) = 3(3x + 7) - 2 = 9x + 21 - 2$$

$$\therefore \quad y = 9x + 19$$

Let $y = 0 \Rightarrow 0 = 9x + 19 \Rightarrow 9x = -19$$

$$\therefore x = -\frac{19}{9} \quad \therefore \quad \left(-\frac{19}{9}, 0\right)$$

**Notation**

$f: x \rightarrow x^2 + 2$ means $f(x) = x^2 + 2$

$x \in \mathbb{R}$

$x$ is a member of the real numbers

$\mathbb{R} = \{\text{real numbers}\}$

$\mathbb{N} = \{\text{natural numbers}\}$

$\mathbb{Q} = \{\text{rational numbers}\}$

$\mathbb{Z} = \{\text{integers}\}$

$\mathbb{C} = \{\text{complex numbers}\}$