Resolving Non-perpendicular Forces

e.g. Find the magnitude and direction of the resultant.

\[ \begin{align*}
\vec{F} &= 10 \text{N} \\
&\quad \downarrow \\
\vec{F} &= 15 \text{N} \\
\vec{F} &= 12 \text{N}
\end{align*} \]

1. Resolve horizontally and vertically

\[ \begin{align*}
(\rightarrow) \quad 15 \cos 40^\circ & - 12 \cos 35^\circ \\
&= 1.6608 \text{N} \\
(\uparrow) \quad 10 & + 15 \sin 40^\circ - 12 \sin 35^\circ \\
&= 12.7589 \text{N}
\end{align*} \]

(Any forces that have a component acting horizontally should be considered)

(Any forces that have a component acting vertically should be considered)

2. Make a force triangle with our resultants to calculate magnitude and direction

\[ F = \sqrt{1.6608^2 + 12.7589^2} \approx 12.87 \text{N} \]

\[ \alpha = \arctan \left( \frac{12.7589}{1.6608} \right) \approx 82.58^\circ \]

\[ \therefore \text{ direction is } 82.58^\circ \text{ above the positive horizontal} \]

Always give a reference line for the directions.
Forces on a Slope

Consider a situation in which a particle is placed at rest on a slope then left to freely roll down the slope.

(No friction mentioned so assume the slope is smooth.)

The slope exerts a force on the particle normal to the slope called the ‘normal reaction force’.

Resolving Forces on a Slope

The direction we should consider when resolving forces on a slope are parallel to the slope (\( R \)) and perpendicular to the slope (\( R \)).

Two components of mg in our chosen direction.

(Note: It is often beneficial to draw a separate diagram with the forces already resolved.)
e.g. The following particle is held at rest. Find the value of $F$ and $R$.

\[ F - 5g \sin 30^\circ = 0 \]
\[ \Rightarrow F = 5g \sin 30^\circ = \frac{5}{2}g \text{ N} \]

\[ R - 5g \cos 30^\circ = 0 \]
\[ \Rightarrow R = 5g \cos 30^\circ = \frac{5\sqrt{3}}{2}g \text{ N} \]

e.g. find the magnitude of $P$ and $Q$ given the system is in equilibrium.

\[ Q + 2\cos 60^\circ - 6 \sin 60^\circ = 0 \]
\[ \Rightarrow Q = 6 \sin 60^\circ - 2 \cos 60^\circ = -1 + 3\sqrt{3} \text{ N} \]

\[ P - 6 \cos 60^\circ - 2 \sin 60^\circ = 0 \]
\[ \Rightarrow P = 6 \cos 60^\circ + 2 \sin 60^\circ = 3 + \sqrt{3} \text{ N} \]
A smooth bead $Y$ is threaded on a light inextensible string. The ends of the string are attached to two fixed points, $X$ and $Z$, on the same horizontal level. The bead is held in equilibrium by a horizontal force of magnitude 8 N acting parallel to $ZX$. The bead $Y$ is vertically below $X$ and $\angle XZY = 30^\circ$ as shown in the diagram.

Find the tension in the string and the weight of the bead.

A single piece of string... tension is the same throughout.

\[ (\uparrow) \ T + T\sin30^\circ - W = 0 \]
\[ \Rightarrow \ T\cos30^\circ - 8 = 0 \]
\[ \Rightarrow \ T = \frac{8}{\cos30^\circ} = \frac{16\sqrt{3}}{3} \text{ N} \]

\[ \Rightarrow \frac{16\sqrt{3}}{3} + \frac{16\sqrt{3}\sin30^\circ}{3} = W = 8\sqrt{3} \text{ N} \]

A mass of 3 kg rests on the surface of a smooth plane which is inclined at an angle of 45° to the horizontal. The mass is attached to a cable which passes up the plane along the line of greatest slope and then passes over a smooth pulley at the top of the plane. The cable carries a mass of 1 kg freely suspended at the other end. The masses are modelled as particles, and the cable as a light inextensible string. There is a force of $PN$ acting horizontally on the 3 kg mass and the system is in equilibrium.

Calculate:  
- a the magnitude of $P$ 
- b the normal reaction between the mass and the plane. 
- c State how you have used the assumption that the pulley is smooth in your calculations.
Resolving at \( A \)

\[ a_1 \begin{align*}
\tau + P \cos 45 - 3g \sin 45 &= 0 \\
\Rightarrow P \cos 45 &= 3g \sin 45 - g \\
\Rightarrow P &= \frac{3g \sin 45 - g}{\cos 45} = 15.54 \text{ N}
\end{align*} \]

\[ b_1 \begin{align*}
R - P \sin 45 - 3g \cos 45 &= 0 \\
\Rightarrow R &= P \sin 45 + 3g \cos 45 = 31.78 \text{ N}
\end{align*} \]

c, Tension is the same throughout the string.