Sketching Families of Solutions to Differential Equations

Example

Sketch the family of solutions to \( \frac{dy}{dx} = xy \)

\[ dy = xy \, dx \implies \frac{1}{y} \, dy = x \, dx \implies \int \frac{1}{y} \, dy = \int x \, dx \]

\[ \implies \ln|y| = \frac{1}{2} x^2 + c \implies y = e^{\frac{1}{2} x^2} = e^{\frac{1}{2} c} = Ae^{\frac{1}{2} x^2} \]

\[ \implies y = Ae^{\frac{1}{2} x^2} \]