Simple Harmonic Motion

Consider a particle moving in a circle of radius r.

If we rotate the circle 90° and view the particle from the side, the particle undergoes motion similar to a pendulum.

In this situation, acceleration is always directed towards the center of the circle.

Definitions (SHM)

Simple harmonic motion is motion in which the acceleration is proportional to the distance from the center of motion.

\[
\frac{d^2x}{dt^2} = -\omega^2 x \quad \text{where} \quad -\omega^2 \text{ is the constant of proportionality}
\]

Example:
Find the general solution \( \ddot{x} = -\omega^2 x \)

Note a dot above a variable indicates \( \frac{dx}{dt} \)

\[
\frac{d^2x}{dt^2} + \omega^2 \frac{dx}{dt} + \omega^2 x = 0
\]

C.F. \( m^2 + \omega^2 = 0 \) \( \Rightarrow m = \pm \omega i \)

\[
\therefore x = A \sin(\omega t) + B \cos(\omega t)
\]

Note that this can easily be transformed to the form

\[
x = R \sin(\omega t + \alpha) \quad \text{or} \quad R \cos(\omega t + \alpha)
\]
$y = A \sin(kx) + B \cos(kx)$

$A = 4.7$

$k = 2.4$

$t = 2.3$

$B = 1$