Writing One Number as a Power of Another

e.g. Write $25^6$ as a power of 5.

Notice that 25 can be rewritten as $5^2$

i.e. $25^6 = (5^2)^6 = 5^{12}$

Both ways $25^6$ in two different ways

e.g. Write $64^5$ as a power of 4

$(4^3)^5 = 4^{15}$

e.g. Write 32 as a power of 4

This is more difficult as 32 is not an integer power of 4. Think of numbers that they both have in common and go into that number.

$4^{\frac{5}{2}} = 2$  \hspace{1cm}  $2^5 = 32$

Using both of these facts

$32 = 2^5 = (4^{\frac{5}{2}})^5 = 4^{\frac{25}{2}}$

e.g. Write 64 in the form $16^n$ where n is rational.

Way 1 :  \hspace{0.5cm} 4 = 16^{\frac{1}{2}}  \hspace{0.5cm} 64 = 4^3  \hspace{0.5cm} 64 = (16^{\frac{1}{2}})^3 = 16^{\frac{3}{2}}$

Way 2 :  \hspace{0.5cm} 2 = 16^{\frac{1}{4}}  \hspace{0.5cm} 64 = 2^6  \hspace{0.5cm} 64 = (16^{\frac{1}{4}})^6 = 16^{\frac{6}{4}} = 16^{\frac{3}{2}}$
Q5 (OCR 4721, Jun 2016, Q5)

Express the following in the form $2^n$.

(i) $(2^5 \div 2^7)^3$

\[
\begin{align*}
\left(2^{-2}\right)^3 &= 2^{-6} \\
5 \cdot 4^{\frac{1}{5}} + 3 \cdot 16^{\frac{1}{3}} &= 5 \cdot (2^\frac{5}{5}) + 3 \cdot (2^4)^{\frac{1}{3}} \\
&= 5 \cdot 2^1 + 3 \cdot 2^{\frac{4}{3}} \\
&= 2^1 \cdot 2^1 + 2^\frac{3}{3} \cdot 2^{\frac{1}{3}} \\
&= 2^{\frac{5}{3}} \\
&= 2^{\frac{3}{3}} \cdot 2^{\frac{1}{3}} \\
&= 2^{\frac{4}{3}}
\end{align*}
\]

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E.g. Solve $2^{x+3} = 4^{3x-2}$

\[
\begin{align*}
2^{x+3} &= (2^2)^{3x-2} \\
&= 2^{6x-4} \\
&= 2 \text{ } \Rightarrow 6x-4 = 2 \\
&= x + 3 = 6x - 4 \\
&= 5x = 7 \Rightarrow x = \frac{7}{5}
\end{align*}
\]

Hint: Write both “big numbers” (also called base numbers) as a power of the same number.

The only way this is possible is if both powers are equal.

E.g. Solve $16^{2x+7} = 8^{5-3x}$

\[
\begin{align*}
16^{2x+7} &= (2^4)^{2x+7} \\
&= 2^{8x+28} \text{ } \Rightarrow 8x + 28 = 15 - 9x \\
&= 17x = -13 \\
&= x = -\frac{13}{17}
\end{align*}
\]