Transformations of graphs and Functions

There are three main types of transformation that can be performed on a function:

- $+$ gives a translation
- $\times$ gives a stretch
- $\ominus$ gives a reflection

There are two rules that need to be learnt when applying these transformations:

1. An "outside" transformation does what it says to $y$-values.
2. An "inside" transformation does the opposite of what it says to $x$-values.

e.g. Describe in words the following transformations applied to $f(x)$.

a/ $f(x) \rightarrow f(x+2)$

Is mapped to $f(x)$ 3 units to the left.

Translate $\left( \begin{array}{c} -2 \\ 0 \end{array} \right)$

b/ $f(x) \rightarrow f(x) + 6$

Translate $\left( \begin{array}{c} 0 \\ 6 \end{array} \right)$

c/ $f(x) \rightarrow f(-x)$

Reflect in y-axis

$y$-axis reflects in $x$-direction
d/ $f(x) \rightarrow 3f(x)$  
Stretch by scale factor 3 parallel to $y$-axis

$x^3$ can outside so
does what 1 says to $y$

e/ $f(x) \rightarrow f(6x)$  
Stretch by scale factor 6 parallel to $x$-axis

f/ $y = x^2 \rightarrow y = x^2 + 4$  
Translate $(0, 4)$
Outside

$y = x^3 \rightarrow y = (-x)^3$  
Reflect in $y$-axis

$h/ y = x^3 \rightarrow y = -x^3$  
Reflect in $x$-axis

e.g. The following graph shows $y = f(x)$.

Sketch the following graphs:

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e.g. Find the equation of the curve when the graph of \( y = x^2 + 2x + 1 \) is transformed as follows.

a/ Translation \((-3,0)\)

\[ y = (x^2 + 2x + 1) - 3 \]
\[ y = x^2 + 2x - 2 \]

b/ Reflection in \( y \)-axis

\[ y = (\pm x)^2 + 2(\pm x) + 1 \]
\[ y = x^2 - 2x + 1 \]
\[ y = (x+3)^4 + 2(x+3) + 1 \]
\[ y = x^2 + 6x + 9 + 2x + 6 + 1 \]
\[ y = x^2 + 8x + 16 \]