Solving Exponential Equations

e.g. \(5^x = 2.8\)

**Problem:** \(x\) is in the power and needs to be "on the line".

\[
\log_5 (5^x) = \log_5 (2.8) \quad \text{Solution: Take logs of both sides}
\]

\[
x = \log_5 (2.8) \\
\approx 0.640 \quad (3sf)
\]

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Alternative solution with different base log.

\[5^x = 2.8\]

\[
\ln(5^x) = \ln(2.8) \quad \text{Power Law}
\]

\[
x \ln(5) = \ln(2.8) \quad \frac{\ln(2.8)}{\ln(5)} \approx 0.640 \quad (3sf)
\]

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e.g. \(2^{x+3} = 7.2\)

\[
\log_2 (2^{x+3}) = \log_2 (7.2)
\]

\[
x + 3 = \log_2 (7.2)
\]

\[
x = \log_2 (7.2) - 3 \approx -0.152 \quad (3sf)
\]

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e.g. \(2^{x+3} = 5^{2x+4}\)

\[
\log (2^{x+3}) = \log (5^{2x+4})
\]

\[
(x+3) \log 2 = (2x+4) \log 5
\]

\[
x \log 2 + 3 \log 2 = 2x \log 5 + 4 \log 5
\]

\[
x \log 2 - 2x \log 5 = 4 \log 5 - 3 \log 2
\]

All of \(2x+4\) is multiplied by \(\log 5\) - need brackets.

Expand brackets
Collecting like terms
\[ x(\log 2 - 2\log 5) = 4\log 5 - 3\log 2 \]

\[ x = \frac{4\log 5 - 3\log 2}{\log 2 - 2\log 5} \approx -1.73 \]