Rationalizing the Denominator of Surds

Surds on denominator of numbers are seen as being bad presentation

\[ \frac{6}{\sqrt{5}} \]

e.g. \[ \frac{6}{\sqrt{5}} \] is BAD!!

However, if we multiply by 1 in a cunning way, we can make the denominator rational.

\[ \frac{6}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{6\sqrt{5}}{5} \]

The RHS is seen as being more acceptable and is called simplified surd form.

e.g. Write \[ \frac{14}{\sqrt{7}} \] in simplified surd form.

\[ \frac{14}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{14\sqrt{7}}{7} = 2\sqrt{7} \]

Like writing \[ \frac{14}{7} \] which can be simplified.

e.g. \[ \frac{7}{\sqrt{6}} \] in simplified surd form

\[ \frac{7}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{35}{150} = \frac{7}{30} \]

Not easiest method

\[ \frac{7}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{7\sqrt{6}}{30} \]
e.g. Simplify \( \frac{\sqrt{45} - 5}{\sqrt{20}} \)

Hint: Any surds that can be simplified should be first to make the solution as easy as possible.

\[
\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5} \quad \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}
\]

\[
\frac{3\sqrt{5} - 5 \times \sqrt{5}}{2\sqrt{5}} = \frac{15 - 5\sqrt{5}}{10} = \frac{3 - \sqrt{5}}{2}
\]

**Denominator of the Form \( a + b\sqrt{c} \)**

The idea of the difference of two squares is extremely useful when rationalizing the denominator of surds.

\[
(3 + \sqrt{2})(3 - \sqrt{2}) = 9 - 3\sqrt{2} + 3\sqrt{2} - 2 = 7
\]

For \( a + b\sqrt{c} \), \( a - b\sqrt{c} \) is called its conjugate.

Applying this to rationalising the denominator:

e.g. give \( \frac{3}{2 - \sqrt{7}} \) a rational denominator.

\[
\frac{3}{2 - \sqrt{7}} \times \frac{2 + \sqrt{7}}{2 + \sqrt{7}} = \frac{6 + 3\sqrt{7}}{4 - 7} = \frac{6 + 3\sqrt{7}}{-3}
\]

Express \( \frac{3 + \sqrt{20}}{3 + \sqrt{5}} \) in the form \( a + b\sqrt{c} \).

\[
= \frac{3 + \sqrt{4} \sqrt{5}}{3 + \sqrt{5}} = \left( \frac{3 + 2\sqrt{5}}{3 + \sqrt{5}} \right) \left( \frac{3 - \sqrt{5}}{3 - \sqrt{5}} \right) = \frac{9 - 3\sqrt{5} + 6\sqrt{5} - 10}{9 - 5} = \frac{-1 + 3\sqrt{5}}{4} = \frac{-1 + \frac{3}{4}\sqrt{5}}{4}
\]