The differential of a function is only easy when the variables used are \( y \) and \( x \), with \( y \) as the subject.

\[ \text{e.g. } y = x^3 + 3x^2 - 2 \implies \frac{dy}{dx} = 3x^2 - 6x \]

If different variables are used, say \( A \) in terms of \( p \), the differential would be written as \( \frac{dA}{dp} \).

\[ \text{e.g. } T = 7s^2 - 3 \implies \frac{dT}{ds} \]

\[ N = 5n^2 + 2n + 2 \implies \frac{dN}{dn} \]

Often in practical situations, the letters are not x and y.

The surface area, \( A \) cm\(^2\), of an expanding sphere of radius \( r \) cm is given by \( A = 4\pi r^2 \). Find the rate of change of the area with respect to the radius at the instant when the radius is 6 cm.

\[ A = 4\pi r^2 \]

\[ \frac{dA}{dr} = 8\pi r \implies \text{at } r = 6, \frac{dA}{dr} = 8\pi (6) = 48\pi \]

A sector of a circle has area 100 cm\(^2\).

a. Show that the perimeter of this sector is given by the formula

\[ P = 2r + \frac{200}{r}, \quad r > \sqrt{\frac{100}{\pi}} \]

b. Find the minimum value for the perimeter.

\[ \frac{1}{\pi r^2} \times \frac{0}{360} = 100 \]

\[ \text{Write out info given in question. This will be used to eliminate a variable.} \]

\[ P = 2r + \ell \]

\[ \text{Write the formula for required quantities.} \]

\[ \ell = \frac{2\pi r \times 0}{360} \]

\[ \implies P = 2r + \frac{2\pi r}{360} \]

\[ \text{Use eqn to eliminate the \textit{unknown variable}} \]

\[ \text{Since } \frac{\pi r \times 0}{360} = 100 \implies \theta = \frac{100 \times 360}{\pi r^2} = \frac{36000}{\pi r^2} \]
\[ P = 2r + 2\pi r \times \frac{36000}{11r^2} \times \frac{1}{360} \]
\[ \Rightarrow P = 2r + 2\pi r \times \frac{100}{11r^2} = 2r + \frac{200}{r} \text{ as required} \]

\[ \frac{dp}{dr} = 2 - \frac{200}{r^2} = 0 \]
\[ \Rightarrow 2 - \frac{200}{r^2} = 0 \Rightarrow 2 = \frac{200}{r^2} \Rightarrow r^2 = 100 \]
\[ \Rightarrow r = \pm 10 \]

\[ r = 10 \]

\[ P = 2(10) + 200(10)^{-1} = 40 \text{ cm} \]

\[ \text{Q4, (Edexcel 6664, Jan 2012, Q8)} \]

Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius \( x \) metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to \( x \) metres and width equal to \( y \) metres.

Given that the area of the flowerbed is 4 m\(^2\),

(a) show that

\[ y = \frac{16 - \pi x^2}{8x} \] \hspace{1cm} (3)

(b) Hence show that the perimeter \( P \) metres of the flowerbed is given by the equation

\[ P = \frac{8}{x} + 2x \] \hspace{1cm} \text{No } y \text{'s}. \text{ use (1) to eliminate } y \text{'s. } \hspace{1cm} (3)

(c) Use calculus to find the minimum value of \( P \). \hspace{1cm} (5)

(d) Find the width of each rectangle when the perimeter is a minimum. Give your answer to the nearest centimetre. \hspace{1cm} (2)
\[ A = \frac{1}{4} \pi r^2 + 2xy = 4 \] 
\[ \pi r^2 + 8xy = 16 \]  
\[ 8xy = 16 - \pi r^2 \]  
\[ y = \frac{16 - \pi r^2}{8x} \]

b/ \[ P = \frac{1}{2} \pi r + 2x + 4y \]  
\[ = \frac{1}{2} \pi r + 2x + 4 \left( \frac{16 - \pi r^2}{2} \right) \]  
\[ = \frac{1}{2} \pi r + 2x + \frac{16}{2} - \frac{\pi r^2}{2} \]  
\[ = \frac{1}{2} \pi r + 2x + \frac{8}{x} - \frac{\pi r^2}{2} = 2x + \frac{8}{x} \]  
\[ \therefore P = \frac{8}{x} + 2x \]

c/ \[ P = 8x^{-1} + 2x \]  
\[ \Rightarrow \frac{dP}{dx} = -8x^{-2} + 2 = 0 \]  
\[ \Rightarrow -8 + 2 = 0 \]  
\[ \Rightarrow -8 = -2 \Rightarrow 8 = 2x^2 \]  
\[ \Rightarrow x^2 = 4 \]  
\[ \Rightarrow x = \pm 2 \]  
\[ \Rightarrow x = 2 \]  
\[ P = 8(2)^{-1} + 2(2) = 8 \]

d/ \[ y = \frac{16 - \pi (2)^2}{8(2)} = 0.2146 \text{ m} \approx 2.1 \text{ cm} \]