**FACT**: Two lines are perpendicular if the product of their gradients is \(-1\). i.e. \(y = m_1 x + c_1\) and \(y = m_2 x + c_2\) are perpendicular if \(m_1 m_2 = -1\) \(\implies m_1 = -\frac{1}{m_2}\)

**Example**: Find the equation of the line that is perpendicular to \(6x + 5y - 3 = 0\) and passes through \((2, -2)\).

**Gradient**: 
\[5y = 3 - 6x \implies y = \frac{3}{5} - \frac{6}{5}x\] 
\[\text{Gradient of the given line} = \frac{-6}{5}\]
\[\text{Gradient of the line} = \frac{5}{6}\] 
**Negative reciprocal to find perpendicular gradient**

**Point**: \((2, -2)\)

\[y - (-2) = \frac{5}{6}(x - 2)\]
\[y + 2 = \frac{5}{6}(x - 2)\]
\[6y + 12 = 5(x - 2)\]
\[6y + 12 = 5x - 10\]
\[5x - 6y - 22 = 0\]
e.g. Find the equation of the perpendicular bisector of \((5,3)\) and \((2,-1)\).

Gradient: \(m = \frac{-1-3}{2-5} = \frac{-4}{-3} = \frac{4}{3}\)

\(\Rightarrow m_b = -\frac{3}{4}\)

Point: M.P. = \(\left(\frac{1}{2}(5+2), \frac{1}{2}(3-1)\right) = \left(\frac{7}{2}, 1\right)\)

Line: \(y - 1 = -\frac{3}{4}(x - \frac{7}{2})\)

\(\Rightarrow 4y - 4 = -3(x - \frac{7}{2})\)

\(\Rightarrow 4y - 4 = -3x + \frac{21}{2}\)

\(\Rightarrow 8y - 8 = -6x + 21\)

\(\Rightarrow 6x + 8y - 29 = 0\)

The points \(A\) and \(B\) have coordinates \((6, 1)\) and \((-2, 7)\) respectively.

(i) Find the length of \(AB\). [2]

(ii) Find the gradient of the line \(AB\). [2]

(iii) Determine whether the line \(4x - 3y - 10 = 0\) is perpendicular to \(AB\). [3]

\(\Rightarrow \left|AB\right| = \sqrt{6^2 + 8^2} = 10\)

\(\Rightarrow m = \frac{7 - 1}{-2 - 6} = \frac{6}{-8} = -\frac{3}{4}\)

\(\Rightarrow 3y = 4x - 10 \quad \Rightarrow \quad y = \frac{4}{3}x - \frac{10}{3}\) which has grad \(\frac{4}{3}\)

\(-\frac{3}{4} \times \frac{4}{3} = -1 \quad \therefore \text{lines are perpendicular.}\)
$A$ is the point $(-2, 6)$ and $B$ is the point $(3, -8)$. The line $l$ is perpendicular to the line $x - 3y + 15 = 0$ and passes through the mid-point of $AB$. Find the equation of $l$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

MP of $AB$: \[\left(\frac{1}{2}(-2+3), \frac{1}{2}(6-8)\right) = \left(\frac{1}{2}, -1\right)\]

Grad of $l$: \[3y = x + 15 \implies y = \frac{1}{3}x + 5\]

$\implies \text{grad of } l = \frac{1}{3}$

$\implies l \perp \text{grad} \implies -3$

$\therefore y - 1 = -3\left(x - \frac{1}{2}\right) \implies y + 1 = -3x + \frac{3}{2}$

$\implies 3x + y - \frac{3}{2} = 0$ \{Must be integer coefficients\}

$\implies 6x + 2y - 1 = 0$