The differential of a function is only easy when the variables used are y and x, with y as the subject.

\[ y = x^3 + 3x^2 - 2 \Rightarrow \frac{dy}{dx} = 3x^2 - 6x \]

If different variables are used, say A in terms of p, the differential would be written as \( \frac{dA}{dp} \).

\[ \text{eg. } T = 7s^2 - 3 \Rightarrow \frac{dT}{ds} \]
\[ N = 5n^2 + 2n + 2 \Rightarrow \frac{dN}{dn} \]

Often in practical situations, the letters are not x and y.

The surface area, \( A \) cm\(^2\), of an expanding sphere of radius \( r \) cm is given by \( A = 4\pi r^2 \). Find the rate of change of the area with respect to the radius at the instant when the radius is 6 cm.

\[ A = 4\pi r^2 \Rightarrow \frac{dA}{dr} = 8\pi r \Rightarrow \text{at } r = 6, \frac{dA}{dr} = 8\pi(6) = 48\pi \]

A sector of a circle has area 100 cm\(^2\).

a. Show that the perimeter of this sector is given by the formula

\[ P = 2r + \frac{200}{r}, \quad r > \sqrt{\frac{100}{\pi}} \]

b. Find the minimum value for the perimeter.

\[ \frac{\pi r^2 \times \theta}{360} = 100 \]

\[ P = 2r + \ell \]

where \( \ell = \frac{2\pi r \times \theta}{360} \)

\[ \Rightarrow P = 2r + \frac{2\pi r \cdot \theta}{360} \]

Since \( \frac{\pi r^2 \times \theta}{360} = 100 \Rightarrow \theta = \frac{100 \times 360}{\pi r^2} = \frac{36000}{\pi r^2} \)
\[ \Rightarrow P = 2r + 2\pi r \times \frac{36000}{11r^2} \times \frac{1}{360} \]

\[ \Rightarrow P = 2r + \frac{200}{r} \quad \text{as required} \]

by \[ P = 2r + 200r^{-1} \]

\[ \frac{dP}{dr} = 2 - 200r^{-2} = 0 \]

\[ \Rightarrow 2 - \frac{200}{r^2} = 0 \quad \Rightarrow \quad 2 = \frac{200}{r^2} \quad \Rightarrow \quad 2r^2 = 200 \]

\[ \Rightarrow r^2 = 100 \quad \Rightarrow \quad r = \pm 10 \]

\[ \boxed{r = 10} \]

\[ P = 2(10) + 200(10)^{-1} = 40 \text{ cm} \]