Proof

Proof is a mathematical tool used to test whether statements are true or not. There are many different types/styles of proof that can be used in different situations.

Proof by Deduction

This is a type of proof used to deduce other true statements from statements we already know to be true.

Notation (\( \Rightarrow \))

This sign means "implies that". It means that the statement on the right of the sign is a direct consequence of the statement on the left.

- e.g. \( x = 3 \Rightarrow x^2 = 9 \)
  - "\( x = 3 \) implies \( x^2 = 9 \)"

- e.g. \( x^2 = 9 \Rightarrow x = 3 \)
  - "\( x^2 = 9 \) does not imply \( x = 3 \)"

This is true because \( x^2 = 9 \Rightarrow x \leq 3 \)

- e.g. \( 2x = 6 \Rightarrow x = 3 \)
  - Also true are \( 2x = 6 \Rightarrow x = 3 \)
  - \( 2x = 6 \Leftarrow x = 3 \)

which allows us to write the double sided implication sign.
Q1. (Jun 2006, Q4)
In each of the following cases choose one of the statements

- $P \Rightarrow Q$
- $P \iff Q$
- $P \Leftarrow Q$

to describe the complete relationship between $P$ and $Q$.

(i) P: $x^2 + x - 2 = 0$  
Q: $x = 1$

\[ (x + 2)(x - 1) = 0 \quad \Rightarrow \quad x = -2 \text{ or } 1 \]
\[ \therefore \quad P \Leftarrow Q \]  

(ii) P: $y^3 > 1$
Q: $y > 1$
\[ \therefore \quad P \iff Q \]

Q4. (Jan 2012, Q9)
Complete each of the following by putting the best connecting symbol ($\iff$, $\Leftarrow$ or $\Rightarrow$) in the box. Explain your choice, giving full reasons.

(i) $n^3 + 1$ is an odd integer
\[ n \text{ is an even integer} \]

(ii) $(x - 3)(x - 2) > 0$
\[ x > 3 \]

\[ \begin{array}{ll}
\therefore \quad n \text{ being even means } n^3 \text{ (Even x Even x Even)} \\
\quad \text{is also even. Add 1 then it becomes odd}
\end{array} \]

\[ \therefore \quad \text{because letting } n^3 + 1 = 7, \text{ we get } n^3 = 6 \]
\[ \text{and } n = \sqrt[3]{6} \text{ which is not even.} \]

\[ \therefore \quad \begin{array}{ll}
\therefore \quad \text{because from the graph we can see that } x > 3 \text{ or } x < 2 \\
\quad \text{satisfy the inequality, not just } x > 3.
\end{array} \]

Proof Involving Odd and Even Numbers

An even number is any integer multiple of 2 (incl. 0).
An odd number is one that is not even (but must be an integer)
Mathematical Definitions
- A number is even if it can be written in the form $2n$ where $n$ is an integer.
- A number is odd if it can be written in the form $2n + 1$ or $2n - 1$ where $n$ is an integer.

E.g. Prove the sum of two even numbers is also even.

Let my even numbers be $2n$ and $2m$

$$2n + 2m = 2(n + m)$$

Exaggerate it being a multiple of 2 by factorising

Which is of the form $2A$ : even

Conclusions is important

Prove that the sum of the squares of any two consecutive numbers is odd.

Let my two numbers be $n, n+1$

$$n^2 + (n+1)^2 = n^2 + (n+1)(n+1)$$

$$= n^2 + n^2 + 2n + 1$$

$$= 2n^2 + 2n + 1 = 2(n^2 + n) + 1$$

which is of the form $2A + 1$ : odd

Important to use two different letters as numbers may be different.