Factoring Quadratics

Factors $x^2 - 6x - 16$

Need two numbers that multiply to make $-16$ and add to make $-6$

Start by listing all factors of $16$ in pairs, ignoring the fact it was $-16$.

See which of these numbers add to make $-6$. The fact that were negative in the quadratic means we use negatives here.

$2 - 8 = -6$

$2, -8$ are the numbers

$\Rightarrow x^2 - 6x - 16 = (x + 2)(x - 8)$

Quadratics with $x^2$ coefficient $\neq 1$

E.g. Factorise $4x^2 + 8x + 3$

Problem is that the brackets could be $(2x \ ) (2x \ )$

$4x \ ) (x \ )$

1. Multiply first and last numbers together.

$4 \times 3 = 12$

Our chosen numbers must multiply to make this number but add to make the middle number.

I.e. in this example $x$ to make $12$ and add to make $8$
2. Find two numbers that meet the above criteria.

6, 2  \quad (\text{i.e.} \quad 6 \times 2 = 12, \quad 6 + 2 = 8)

3. Write the quadratic out again but this time split the \('x\) term into two parts using the pair of numbers you have just found.

\[ 4x^2 + 8x + 3 \]

\[ \underline{4x^2 + 6x} \underline{+ 2x + 3} \]

4. Fully factorise each pair of terms.

\[ 2x(2x + 3) + 1(2x + 3) \]

Notice the brackets contain the same expression. This reassures us we are right.

5. Factorise again.

\[ (2x + 3)(2x + 1) \]

\[ \text{e.g. factorise} \quad 6p^2 + 5p - 1 \]

\[ 6 \times -1 = -6 \]

\[ \times \text{ to get} -6 \]

\[ + \text{ to get} 5 \]

\( \left\{ \begin{array}{l}
6, -1
\end{array} \right. \)

\[ \frac{6p^2 + 6p - p - 1}{\underline{6p} \underline{(p + 1)} - 1(p + 1)} \]

\[ = (p + 1)(6p - 1) \]
e.g. Factorize \( 8x^2 + 19x + 6 \)

\[
8 \times 6 = 48
\]

\[
\begin{array}{c}
\times \text{ to get } 48
\hline
+ \text{ to get } 19
\end{array}
\]

\[
\begin{array}{c}
(3, 16)
\hline
(5, 16)
\end{array}
\]

\[
8x^2 + 3x + 16x + 6
\]

\[
=x(8x + 3) + 2(8x + 3)
\]

\[
=(8x + 3)(x + 2)
\]

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Solving Equations by Factorizing

\[
6 + 23x - 4x^2 = 0
\]

\[
\begin{array}{c}
-4x^2 + 23x + 6 = 0
\hline
4x^2 - 23x - 6 = 0
\end{array}
\]

\[
\begin{array}{c}
\text{Let } 4x^2 - 23x - 6 = 0
\hline
\text{Factorization: } 4x(3x - 6) + 4(x - 6) = 0
\end{array}
\]

\[
\Rightarrow (x - 6)(4x + 4) = 0
\]

\[
\Rightarrow 4x + 4 = 0 \quad \text{or} \quad x - 6 = 0
\]

\[
\begin{array}{c}
4x + 4 = 0
\hline
x - 6 = 0
\end{array}
\]

\[
\begin{array}{c}
4x = -4
\hline
x = 6
\end{array}
\]

\[
\begin{array}{c}
\text{Two things } x \text{ together to make } 0
\hline
\text{One of these (or both) must be } 0
\end{array}
\]

\[
\Rightarrow x = -\frac{1}{4}
\]

\[
\begin{array}{c}
x = 6
\hline
4x = -1
\end{array}
\]

\[
x = \frac{-1}{4}
\]
Calculator Instructions

1. Go to equation solving mode

2. Input coefficients (use $4x^2 - 23x - 6 = 0$ as an example)