Surd

A surd is a number that cannot be expressed without a root sign (i.e., \( \sqrt{}, \sqrt[3]{}, \sqrt[n]{}, \) etc).

**Fact:** \( \sqrt{p} \) where \( p \) is prime is a surd

Surds are irrational numbers, i.e., cannot be written as a ratio of integers \( \frac{a}{b} \) where \( a, b \) are integers \((a, b \in \mathbb{Z})\).

**Adding/Subtracting Surds**

We treat surds like we treat unknowns in algebra.

\[
\begin{align*}
3\sqrt{5} + 2\sqrt{5} &= 5\sqrt{5} \\
5\sqrt{2} - 3\sqrt{3} &\text{ cannot be simplified} \\
5\sqrt{2} - 3\sqrt{3} &\text{ cannot be simplified}
\end{align*}
\]

**Multiplying Surds**

Surds multiply like other surds as you would expect.

\[
\begin{align*}
\sqrt{2} \times \sqrt{3} &= \sqrt{6} \\
\sqrt{7} \times \sqrt{3} &= \sqrt{21}
\end{align*}
\]

**Common Mistake**

\[ 2 \times \sqrt{3} = \sqrt{6} \times \]

The above is wrong because both numbers must be written as surds in order to multiply them together.

Since \( 2 = \sqrt{4} \), the above becomes:

\[
\sqrt{4} \times \sqrt{3} = \sqrt{12}
\]

Notice that the calculator presents surds in simplified surd form.
Simplified Surd Form

In order to write surds in simplified surd form, we must first break down the surd into prime surd factors, extract any integers, then rebuild the remaining primes.

\[ \sqrt{45} = \sqrt{9 \times 5} = \sqrt{9} \sqrt{5} = 3\sqrt{5} \]

Could have spotted \( \sqrt{9} = 3 \) at this stage.

\[ \sqrt{42} = \sqrt{6 \times 7} = \sqrt{2 \times 3 \times 7} \]

\[ = \sqrt{2} \times \sqrt{3} \times \sqrt{7} = \sqrt{42} \]

No integers can be obtained, therefore we must recombine the primes.

This was already in simplified surd form.