Discrete Random Variable

A discrete value is one that can only take a certain number of fixed values (i.e., they can be counted).

A variable is random if its value depends on chance. Something can be described as random even if there is not an equal chance of each outcome.

e.g. an unfair dice, whilst biased is still random.

Probability Distribution

A probability distribution is a list or a formula that describes all possible outcomes of a random variable $X$ and the associated probabilities.

e.g. the following table is the probability distribution for an unfair dice.

<table>
<thead>
<tr>
<th>$X$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X=x)$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{2}{6}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
</tr>
</tbody>
</table>

Large $X$ for ‘actual outcome’

Small $x$ for ‘possible outcome’

e.g. A biased dice has a probability of $\frac{3}{4}$ of obtaining a 6. All other outcomes are equally likely. Write the probability distribution of $X$, where $X$ represents the number rolled.

<table>
<thead>
<tr>
<th>$X$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X=x)$</td>
<td>$\frac{1}{20}$</td>
<td>$\frac{1}{20}$</td>
<td>$\frac{1}{20}$</td>
<td>$\frac{1}{20}$</td>
<td>$\frac{3}{4}$</td>
<td>$\frac{1}{4}$</td>
</tr>
</tbody>
</table>

$1 - \frac{3}{4} = \frac{1}{4}$

$\frac{1}{4} \div 5 = \frac{1}{20}$
Expectation and Variance of a D.R.V.

Expectation in this context means 'mean score per turn'.

Take for example, the previous distribution

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X=x)</td>
<td>( \frac{1}{20} )</td>
<td>( \frac{1}{20} )</td>
<td>( \frac{1}{20} )</td>
<td>( \frac{1}{20} )</td>
<td>( \frac{1}{4} )</td>
<td></td>
</tr>
</tbody>
</table>

- \( E(X) \) means 'expectation' or 'mean per throw' of \( X \)
- \( E(X) = \sum x \cdot p \)
- In this example \( E(X) = \frac{1}{20} + 2 \cdot \frac{1}{20} + 3 \cdot \frac{1}{20} + 4 \cdot \frac{1}{20} + 5 \cdot \frac{1}{20} + 6 \cdot \frac{3}{4} \)

\[ = \frac{21}{4} = 5.25 \]

- \( E(X^2) \) means the expectation or mean of all the \( X^2 \) values
- \( E(X^2) = \sum x^2 \cdot p \)

In the above, \( E(X^2) = \frac{1 \cdot 1}{20} + 2^2 \cdot \frac{1}{20} + 3^2 \cdot \frac{1}{20} + 4^2 \cdot \frac{1}{20} + 5^2 \cdot \frac{1}{20} + 6^2 \cdot \frac{3}{4} \)

\[ = 29.75 \]

\( \sigma^2 = \bar{X^2} - \bar{X}^2 \)

\( \text{Vor}(X) = E(X^2) - E(X)^2 \)

In the above, \( \text{Vor}(X) = 29.75 - 5.25^2 \approx 1.48 \)

This measures what we expect the variance of a set of data taken from a large number of throws to be.