The $X^2$ test can be used to test how well a distribution fits a data set.

Eggs are sold in four categories: small, medium, large and extra large. A supermarket model predicts that these will be sold in the ratio $1:2:3:1$. To check this model the supermarket looks at sales in a store in one day.

<table>
<thead>
<tr>
<th>Size of eggs</th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
<th>Extra large</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number sold</td>
<td>16</td>
<td>17</td>
<td>24</td>
<td>13</td>
</tr>
</tbody>
</table>

Use an appropriate statistical test to determine if the model fits this data, using a 5% significance level.

1. **State hypotheses**
   
   $H_0$: Eggs are sold in the ratio $1:2:3:1$
   
   $H_1$: Eggs are not sold in the ratio $1:2:3:1$

2. **Calculate expectations by splitting total into the given ratio**
   
   Total = 16 + 17 + 24 + 13 = 70
   
   $70$ in the ratio $1:2:3:1$
   
   $\frac{70}{7} = 10 \Rightarrow \text{ratio} = 10:20:30:10$

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</tr>
</thead>
<tbody>
<tr>
<td>Number sold</td>
<td>16</td>
<td>17</td>
<td>24</td>
<td>13</td>
</tr>
<tr>
<td>Expected</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>10</td>
</tr>
</tbody>
</table>

3. **Perform a $X^2$ test on the differences.** Note that for goodness of fit tests, even when $v = 1$ a Yates’ correction in never required.
   
   Contributions:
   
   $\frac{(16-10)^2}{10} = 3.6$  
   $\frac{(17-20)^2}{20} = 0.45$  
   $\frac{(24-30)^2}{30} = 1.2$  
   $\frac{(13-10)^2}{10} = 0.9$

   See below for instructions.
Goodness of Fit on Graphical Calculator

A) Input observed and expected data

B) Input observed and expected data

C) Input observed and expected data

D) Input degrees of freedom. In Year 1, the only constraint in the total therefore
   \[ v = n^2 \text{ of items} - 1 \]

E) Input degrees of freedom. In Year 1, the only constraint in the total therefore
   \[ v = n^2 \text{ of items} - 1 \]

F) Pressing "EXIT" twice gives us data, including the contributions in List 3

G) \[ x^2 \text{ GOF Test} \]

H) \[ x^2 = 6.16 \]

I) \[ p = 0.10459613 \]

J) \[ df = 3 \]

K) \[ \text{CNTRB: List 3} \]

L) \[ \text{List 3} \]

M) \[ \text{List 4} \]

N) Conclude

<table>
<thead>
<tr>
<th>( p )</th>
<th>0.01</th>
<th>0.025</th>
<th>0.05</th>
<th>0.90</th>
<th>0.95</th>
<th>0.975</th>
<th>0.99</th>
<th>0.995</th>
<th>0.999</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v = 1 )</td>
<td>0.81571</td>
<td>0.959821</td>
<td>0.959932</td>
<td>2.706</td>
<td>3.841</td>
<td>5.024</td>
<td>6.635</td>
<td>7.879</td>
<td>10.83</td>
</tr>
<tr>
<td>2</td>
<td>0.02010</td>
<td>0.05064</td>
<td>0.1026</td>
<td>4.605</td>
<td>5.991</td>
<td>7.378</td>
<td>9.210</td>
<td>10.60</td>
<td>13.82</td>
</tr>
<tr>
<td>3</td>
<td>0.11480</td>
<td>0.21584</td>
<td>0.3518</td>
<td>6.251</td>
<td>7.815</td>
<td>9.348</td>
<td>11.34</td>
<td>12.84</td>
<td>16.27</td>
</tr>
</tbody>
</table>

6.15 < 7.815

Do not reject \( H_0 \)

Insufficient evidence to suggest that the ratio of eggs differs from 1:2:3:1