

Binomial Hypothesis Testing Problems Longer Exam Questions MS

Q1 (Jun 2016, Q7)

(i)	(A)	$X \sim B(16, 0.1)$ $P(X = 3) = 0.25^4 \times 0.75^{16} \binom{20}{4} \times 0.25^4 \times 0.75^{16} = 0.1423$ Or: From tables $P(X \leq 3) - P(X \leq 2) = 0.9316 - 0.7892 = 0.1424$	M1 M1 A1 M2 A1 [3]	For $0.1^3 \times 0.9^{13}$ For $\binom{16}{3} \times p^3 \times q^{13}$ CAO For $0.9316 - 0.7892$ CAO	With $p + q = 1$ Also for $560 \times 0.000254..$ Allow 0.14 or better
(i)	(B)	$P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0.7892 = 0.2108$	M1 A1 [2]	For 0.7892 CAO	If calculating $P(X = 0) + P(X = 1) + P(X = 2)$ allow M1 for 0.79 or better and A1 for 0.21 or better.

(i)	(C)	Expected number = $16 \times 0.1 = 1.6$	B1 [1]		Do not allow final answer of 1 or 2 even if correct 1.6 given earlier
(ii)		Let p = probability of a randomly chosen person using 1234 as their PIN (in the population) $H_0: p = 0.1$ $H_1: p < 0.1$ The alternative hypothesis has this form as the advertising campaign aims to reduce the proportion of the population who use 1234 as their PIN.	B1 B1 B1 B1 [4]	For definition of p (in context) For H_0 For H_1 Dep on < 0.1 used in H_1 Do Not allow just 'proportion will be lower' or similar.	Do NOT allow <u>number</u> in place of probability. See below for additional notes For use of 0.9 as P(do not use 1234), contact team leader. E0 for simply stating H_1 in words Condone number instead of proportion.
(iii)	(A)	For $n = 20$, $P(X \leq 0) = 0.1216$ $0.1216 > 0.10$ So no point in carrying out the test as H_0 could not be rejected (even if nobody in the sample uses 1234 as their PIN). oe	M1* *M1dep A1 [3]	For sight of 0.1216 For > 0.10 or $> 10\%$ Do NOT FT wrong H_1 or state 'There is no critical region' oe For A1 need $P(X \leq 0)$ or $P(X = 0)$ somewhere oe	Condone $P(X = 0)$ in place of $P(X \leq 0)$ Need to see a comparison with 0.1 or 10% explicitly, not just mentioning significance level. Allow SC2 for clearly indicating use of $B(20, 0.1)$ but with no mention of 0.1216 with convincing reasoning and final answer correct Allow CR is empty but NOT CR is zero
	(B)	Lowest value of k is 13	B1 [1]	Or 13%	
(iv)		$P(X \leq 2) = 0.0530$ $0.0530 > 0.05$ So not significant. Do not reject H_0 Conclude that there is not enough evidence to support the suggestion that the advertising campaign has been successful. Reminder: When you mark this question part, if you 'fit to height' you can check the last page for working or mark it BP if there none	B1 M1 A1* *E1 dep [4]	For <u>use</u> of $P(X \leq 2)$ only For comparison of 0.0530 with 5% Also allow $P(X \leq 2) > 0.05$, $(P(X \leq 1) < 0.05)$ so CR is $\{0, 1\}$ for first two marks then A1E1 as usual Condone 'number of people' in conclusion	No marks unless H_1 correct If B0 then no further marks Allow 'accept H_0 ' or 'reject H_1 ' Must include 'insufficient evidence to suggest that' or something similar i.e. an element of doubt either in the A or E mark.

Q2 (Jun 2015, Q7)

(i)	(B)	$P(\text{Exactly 20 cured}) = \binom{20}{20} \times 0.78^{20} \times 0.22^0 = 0.0069$ $P(\text{At most 18 cured}) = 1 - (0.0069 + 0.0392)$ $= 0.954 \text{ (0.95385)}$	<p>M1</p> <p>M1</p> <p>A1 [3]</p>	<p>For 0.78^{20} oe</p> <p>For $P(19) + P(20)$</p> <p>CAO</p>	<p>Allow M2 for 0.9488 for linear interpolation from tables or M1 for $1 - 0.9918 = 0.0082$ and second M1 for correct FT using answer to (i)(A)</p> <p>Zero for use of $p = 0.8$ here</p> <p>Not necessarily correct, but both attempts at binomial, including coefficient in (i) and no extra terms (such as $P(X=18)$)</p> <p>Condone use of $p = 0.8$</p> <p>Allow 0.95 with working</p>
(i)	(C)	$E(X) = np = 20 \times 0.78 = 15.6$	<p>B1</p> <p>[1]</p>	<p>CAO</p>	<p>Do not allow final answer of 15 or 16 even if correct 15.6 given earlier</p>
(ii)		<p>Let $X \sim B(20, 0.78)$</p> <p>Let p = probability of a patient being cured (for population)</p> <p>$H_0: p = 0.78$</p> <p>$H_1: p > 0.78$</p>	<p>B1</p> <p>B1</p> <p>B1</p>	<p>For definition of p</p> <p>For H_0</p> <p>For H_1</p>	<p>In context</p> <p>See below for additional notes</p> <p>No further marks if point probabilities</p>

<p>$P(X \geq 19) = 0.0392 + 0.0069$</p> <p>$= 0.0461$</p> <p>$0.0461 > 1\%$</p> <p>So not significant. Conclude that there is not enough evidence to suggest that the new drug is more effective than the old one.</p>	<p>B1</p> <p>B1*</p> <p>M1* dep A1 E1</p> <p>[8]</p>	<p>For NOTATION $P(X \geq 19)$ or $P(X > 18)$ or $1 - P(X \leq 18)$ or $1 - P(X < 19)$</p> <p>CAO For 0.0461 allow 0.0462</p> <p>For comparison with 1%</p>	<p>used Notation $P(X = 19)$ scores B0. If they have the correct $P(X \geq 19)$ then give B1 and ignore any further incorrect notation.</p> <p>FT answer to (i)B for following three marks provided based on $1 - (P(19) + P(20))$ Dep on sensible attempt at $P(X \geq 19)$</p> <p>Allow 'accept H_0' or 'reject H_1' Must include 'insufficient evidence' or something similar such as 'to suggest that' ie an element of doubt either in the A or E mark. Must be in context to gain E1 mark. Do NOT allow 'sufficient evidence to suggest proportion cured is 0.78' or similar <u>99% method:</u> $P(X \leq 18) = 0.9539$ B1B1* CAO $0.9539 < 99\%$ M1* then as per scheme</p>
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Q3 (Jun 2014, Q7)

(i)	(A)	$X \sim B(15, 0.85)$	M1	For $0.85^{12} \times 0.15^3$	
		$P(\text{exactly 12 germinate}) = \binom{15}{12} \times 0.85^{12} \times 0.15^3$ $= 0.2184$	M1	For $\binom{15}{12} \times p^{12} \times q^3$	
		OR	OR		
		from tables: $0.3958 - 0.1773$ $= 0.2185$	M2	For $0.3958 - 0.1773$	
			A1	CAO	
			[3]		
(i)	(B)	$P(X < 12) = P(X \leq 11) = 0.1773$	M1	For $P(X \leq 11)$ or $P(\leq 11)$ (With no extras) CAO (as final answer) May see alternative method: $0.3958 - 0.2185 = 0.1773$ 0.3958 - their wrong answer to part (i) scores M1A0	
			A1		
			[2]		

(ii)	<p>Let p = probability of a seed germinating (for the population)</p> <p>$H_0: p = 0.85$ $H_1: p < 0.85$</p> <p>H_1 has this form because the test is to investigate whether the proportion of seeds which germinate is lower.</p>	<p>B1 B1 B1 E1 [4]</p>	<p>For definition of p</p> <p>For H_0</p> <p>For H_1</p> <p>Dep on < 0.85 used in H_1 Do not allow just 'Germination rate will be lower' or similar.</p>	<p>See below for additional notes</p> <p>For use of 0.15 as P(not germinating), contact team leader E0 for simply stating H_1 in words</p>
(iii)	<p>Let $X \sim B(20, 0.85)$ $P(X \leq 13) = 0.0219$</p> <p>$0.0219 > 1\%$</p> <p>So not enough evidence to reject H_0. Not significant.</p> <p>Conclude that there is not enough evidence to indicate that the proportion of seeds which have germinated has decreased.</p>	<p>M1* M1* dep A1* E1* dep</p>	<p>For probability (provided not as part of finding $P(X = 13)$) Ignore notation</p> <p>For comparison</p> <p>For not significant oe</p> <p>For conclusion in context Must mention decrease, not just change</p>	<p>No further marks if point probs used - $P(X = 13) = 0.0160$ DO NOT FT wrong H_1, but see extra notes Allow 'accept H_0' or 'reject H_1' Must include 'sufficient evidence' or something similar such as 'to suggest that' ie an element of doubt either in the A or E mark.</p>

(iv)	<p>$33 < 35$</p> <p>So there is sufficient evidence to reject H_0</p> <p>Conclude that there is enough evidence to indicate that the proportion of seeds which have germinated has decreased.</p>	<p>M1</p> <p>A1*</p> <p>E1* dep</p> <p>[3]</p>	<p>For comparison</p> <p>For conclusion in context</p> <p>Must mention decrease, not just change</p>	<p>Allow '33 lies in the CR' Must include 'sufficient evidence' or something similar such as 'to suggest that' ie an element of doubt either in the A or E mark.</p> <p>Do not FT wrong H_1: In part (iv) ignore any interchanged H_0 and H_1 seen in part (ii)</p> <p>If use a calculator to find $P(X \leq 33) = 0.000661$ and compare with 1% then B2 for $P(X \leq 33) = 0.000661 < 0.01$ so reject H_0 then final E1 as per scheme.</p>
(v)	<p>For $n = 3$, $P(X \leq 0) = 0.0034 < 0.01$ For $n = 2$, $P(X \leq 0) = 0.0225 > 0.01$</p> <p>So the least value of n for which the critical region is not empty and thus H_0 could be rejected is 3.</p> <p>ALTERNATIVE METHOD using logs $0.15^n < 0.01$ $n > \log 0.01 / \log 0.15$ $n > 2.427$ Least $n = 3$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>For $P(X \leq 0) = 0.0034$ For $P(X \leq 0) = 0.0225$</p> <p>CAO</p>	<p>Allow 0.003</p> <p>Condone just '$n = 3$' for final A mark dep on both M marks</p> <p>If wrong H_1 allow max M2A0 if correct probabilities seen.</p>

Q4 (Jan 2013, Q7)

(i)	(A)	$X \sim B(10, 0.35)$ $P(5 \text{ accessing internet}) = \binom{10}{5} \times 0.35^5 \times 0.65^5$ $= 0.1536$ OR from tables $= 0.9051 - 0.7515 = 0.1536$	M1 M1 A1 OR M2 A1 [3]	or $0.35^5 \times 0.65^5$ For $\binom{10}{5} \times p^5 \times q^5$ cao For $0.9051 - 0.7515$ cao	With $p + q = 1$ Also for 252×0.0006094 Allow 0.15 or better NB 0.153 gets A0 See tables at the website http://www.mei.org.uk/files/pdf/formula_book_mf2.pdf
(i)	(B)	$P(X \geq 5) = 1 - P(X \leq 4)$ $= 1 - 0.7515$ $= 0.2485$	M1 A1 [2]	For 0.7515 cao	Accept 0.25 or better – allow 0.248 or 0.249 Calculation of individual probabilities gets B2 if fully correct 0.25 or better, otherwise B0.
(i)	(C)	$E(X) = np = 10 \times 0.35$ $= 3.5$	M1 A1 [2]	For 10×0.35 cao	If any indication of rounding to 3 or 4 allow M1A0

(ii)	<p>Let $X \sim B(20, 0.35)$ Let p = probability of a customer using the internet (for population)</p>	B1	For definition of p in context	Minimum needed for B1 is p = probability of using internet. Allow $p = P(\text{using internet})$ Definition of p must include word probability (or chance or proportion or percentage or likelihood but NOT possibility). Preferably as a separate comment. However can be at end of H_0 as long as it is a clear definition 'p = the probability of using internet', Do NOT allow 'p = the probability of using internet is different'
	<p>$H_0: p = 0.35$</p>	B1	For H_0	Allow $p=35\%$, allow only p or θ or π or ρ . However allow any single symbol <u>if defined</u> (including x) Allow $H_0 = p=0.35$, Allow $H_0: p=7/20$ or $p=35/100$ Allow NH and AH in place of H_0 and H_1 Do not allow $H_0: P(X=x) = 0.35$ Do not allow $H_0: =0.35, =35\%, P(0.35), p(x)=0.35, x=0.35$ (unless x correctly defined as a probability) Do not allow H_0 and H_1 reversed For hypotheses given in words allow Maximum B0B1B1 Hypotheses in words must include probability (or chance or proportion or percentage) and the figure 0.35 or Thus eg $H_0: p(\text{using internet}) = 0.35$, $H_1: p(\text{using internet}) \neq 0.35$ gets B0B1B1

<p>$H_1: p \neq 0.35$</p> <p>H_1 has this form because the test is to investigate whether the proportion is different, (rather than lower or higher). $P(X \geq 10)$</p> <p>$= 1 - 0.8782 = 0.1218$</p> <p>$> 2.5\%$</p> <p>So not significant. Conclude that there is not enough evidence to indicate that the probability is different. (Must state 'probability', not just 'p')</p>	<p>B1</p> <p>E1</p> <p>B1</p> <p>B1*</p> <p>M1*</p> <p>dep</p> <p>A1*</p> <p>E1*</p> <p>dep on</p> <p>A1</p>	<p>For H_1</p> <p>For notation $P(X \geq 10)$ or $P(X > 9)$ or $1 - P(X \leq 9)$ (as long as no incorrect notation)</p> <p>For 0.1218 Allow 0.12</p> <p>For comparison with 2.5%</p>	<p>Allow '$p < 0.35$ or $p > 0.35$' in place of $p \neq 0.35$ Do not allow if H_1 wrong.</p> <p>This mark may be implied by 0.1218 as long as no incorrect notation. No further marks if point probs used - $P(X = 10) = 0.0686$ (do not even give the notation mark for correct notation) DO NOT FT wrong H_1, but see extra notes Or for $1 - 0.8782$ Indep of previous mark</p> <p>Allow 'accept H_0' or 'reject H_1' Must include 'sufficient evidence' or something similar such as 'to suggest that' ie an element of doubt either in the A or E mark.</p>
<p>(iii)</p>	<p>$0.0022 < 2.5\%$ So reject H_0, Significant.</p> <p>Conclude that there is enough evidence to indicate that the probability is different.</p>	<p>B1</p> <p>E1*</p> <p>dep</p> <p>[2]</p>	<p>For either reject H_0 or significant, dep on correct comparison</p> <p>Dep on good attempt at correct hypotheses in part (ii)</p> <p>If they have $H_1: p > 0.35$, allow SC1 if all correct including comparison with 5%.</p>

Q5 (Jun 2013, Q5)

(i)		<p>Because if people cannot make a correct identification, then the probability that they guess correctly will be 0.5 For 'equally likely to guess right or wrong' or 'two outcomes with equal probability' or '50:50 chance of success' or 'right one in two occasions on average' or 'two (equally likely) outcomes' etc</p>	<p>E1 E1 [2]</p>	<p>For idea of a guess or 'chosen at random' For idea of two outcomes</p>	<p>NB The question includes the sentence 'She suspects that people do no better than they would by guessing.', so this on its own does not get the mark for the idea of a guess</p>
(ii)		<p>'Because people may do better than they would by guessing' or similar</p>	<p>B1 [1]</p>	<p>For idea of selecting correctly /identifying /knowing</p>	<p>No marks if answer implies that it is because there are over half in the sample who make a correct identification</p>
(iii)		<p>$P(X \geq 13) = 1 - P(X \leq 12) = 1 - 0.8684 = 0.1316$</p> <p>NB PLEASE ANNOTATE THE TOP AND BOTTOM OF THE EXTRA PAGE IF NOT USED</p> <p>$0.1316 > 0.05$</p> <p>So not significant</p> <p>There is insufficient evidence to suggest that people can make a correct identification.</p>	<p>M1 B1* M1* dep A1* E1* dep</p>	<p>For notation $P(X \geq 13)$ or $P(X > 12)$ or $1 - P(X \leq 12)$ For 0.1316 For comparison with 5%</p> <p>NB Point probabilities score zero.</p>	<p>Notation $P(X = 13)$ scores M0. If they have the correct $P(X \geq 13)$ then give M1 and ignore any further incorrect notation. Or for $1 - 0.8684$ indep of previous mark</p> <p>Allow 'accept H_0' or 'reject H_1'</p> <p>Must include 'insufficient evidence' or something similar such as 'to suggest that' ie an element of doubt either in the A or E mark. Must be in context to gain E1 mark. Do not allow 'sufficient evidence to suggest proportion making correct identification is 0.5' or similar</p>

Q6 (Jun 2012, Q5)

<p>Let p = probability that a randomly selected frame is faulty</p> <p>$H_0: p = 0.05$</p> <p>$H_1: p > 0.05$ $P(X \geq 4)$</p> <p>$= 1 - P(X \leq 3) = 1 - 0.9891 = 0.0109$</p>	<p>B1 For definition of p in context Minimum needed for B1 is p = probability that frame/bike is faulty. Do not allow is p = probability that it is faulty Allow $p = P(\text{frame faulty})$ Definition of p must include word probability (or chance or proportion or percentage or likelihood but NOT possibility). Preferably as a separate comment. However can be at end of H_0 as long as it is a clear definition 'p = the probability that frame is faulty, NOT just a sentence 'probability is 0.05' Do NOT allow 'p = the probability that faulty frames have increased'</p> <p>B1 $H_0: p(\text{frame faulty}) = 0.05, H_1: p(\text{frame faulty}) > 0.05$ gets B0B1B1 Allow $p=5\%$, allow θ or π and ρ but not x. However allow any single symbol if defined Allow $H_0 = p=0.05$, Allow $H_0: p=1/20$ Do not allow $H_0: P(X=x) = 0.05, H_1: P(X=x) > 0.05$ Do not allow $H_0: =0.05, =5\%, P(0.05), p(0.05), p(x)=0.05, x=0.05$ (unless x correctly defined as a probability) Do not allow $H_1: p \geq 0.05$, Do not allow H_0 and H_1 reversed Allow NH and AH in place of H_0 and H_1 For hypotheses given in words allow Maximum B0B1B1 Hypotheses in words must include probability (or chance or proportion or percentage) and the figure 0.05 oe.</p>	<p>B1</p> <p>B1 For notation $P(X \geq 4)$ or $1 - P(X \leq 3)$ This mark may be implied by 0.0109 as long as no incorrect notation.</p> <p>B1* For 0.0109, indep of previous mark</p> <p>No further marks if point probs used - $P(X = 4) = 0.0094$ DO NOT FT wrong H_1 But if H_1 is $p \geq 0.05$ allow the rest of the marks if earned so max 7/8 Or for $1 - 0.9891$</p>
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<p>$0.0109 < 0.05$</p> <p>So reject H_0</p> <p>There is evidence to suggest that the proportion of faulty frames has increased.</p> <p>OR Critical region method: Let $X \sim B(18, 0.05)$ $P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0.9419 = 0.0581 > 5\%$ $P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.9891 = 0.0109 < 5\%$</p> <p>So critical region is $\{4,5,6,7,8,9,10,11,12,13,14,15,16,17,18\}$ 4 lies in the critical region, so significant,</p> <p>There is evidence to suggest that the proportion of faulty frames has increased.</p>	<p>M1* dep A1*</p> <p>E1* Dep on A1</p> <p>[8]</p> <p>(B1)</p> <p>(B1)</p> <p>(M1)</p> <p>(A1)</p> <p>(E1)</p>	<p>For comparison with 5% or significant or 'accept H_1'</p> <p>Must include 'sufficient evidence' or something similar such as 'to suggest that' ie an element of doubt for E1. 'Sufficient evidence' or similar can be seen in the either the A mark or the E mark.</p> <p>For 0.0581</p> <p>For 0.0109</p> <p>For at least one correct comparison with 5%</p> <p>CAO for critical region and significant oe</p>	<p>No marks if CR not justified Do not insist on correct notation as candidates have to work out two probabilities for full marks</p> <p>Condone $\{4,5 \dots\}$, $X \geq 4$, oe but not $P(X \geq 4)$</p>
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