Trigonometric Equations and Identities Exam Questions (From OCR 4722)

Q1, (Jun 2012, Q7a)

(i) Given that α is the acute angle such that $\tan\alpha = \frac{2}{5}$, find the exact value of $\cos\alpha$. **[2]**

(ii) Given that β is the obtuse angle such that $\sin\beta = \frac{3}{7}$, find the exact value of $\cos\beta$. **[3]**

Q2, (OCR 4752, Jun 2006, Q3)

θ is an acute angle and $\sin\theta = \frac{1}{4}$. Find the exact value of $\tan\theta$. [3]

Q3, (OCR 4752, Jan 2007, Q3)

Given that $\cos\theta = \frac{1}{3}$ and θ is acute, find the exact value of $\tan\theta$. [3]

Q4, (OCR 4752, Jan 2008, Q3)

You are given that $\tan\theta = \frac{1}{2}$ and the angle θ is acute. Show, without using a calculator, that $\cos^2\theta = \frac{4}{5}$. [3]

Q5, (Jan 2010, Q1)

(i) Show that the equation

$$2\sin^2 x = 5\cos x - 1$$

can be expressed in the form

$$2\cos^2 x + 5\cos x - 3 = 0.$$ **[2]**

(ii) Hence solve the equation

$$2\sin^2 x = 5\cos x - 1,$$

giving all values of x between 0° and 360°. **[4]**

Q6, (Jun 2010, Q7)

(i) Show that $\dfrac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} \equiv \tan^2 x - 1$. **[2]**

(ii) Hence solve the equation

$$\frac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} = 5 - \tan x,$$

for $0° \leqslant x \leqslant 360°$. **[6]**

Q7, (Jun 2013, Q2)

Solve each of the following equations, for $0° \leqslant x \leqslant 360°$.

(i) $\sin\frac{1}{2}x = 0.8$ **[3]**

(ii) $\sin x = 3\cos x$ **[3]**

Q8, (Jun 2009, Q5)

Solve each of the following equations for $0° \leqslant x \leqslant 180°$.

(i) $\sin 2x = 0.5$ **[3]**

(ii) $2\sin^2 x = 2 - \sqrt{3}\cos x$ **[5]**

Q9, (Jun 2014, Q4)

(i) Show that the equation

$$\sin x - \cos x = \frac{6\cos x}{\tan x}$$

can be expressed in the form

$$\tan^2 x - \tan x - 6 = 0.$$ **[2]**

(ii) Hence solve the equation $\sin x - \cos x = \frac{6\cos x}{\tan x}$ for $0° \leqslant x \leqslant 360°$. **[4]**

Q10, (Jun 2017, Q9) [Modified]

The cubic polynomial f(x) is defined by $f(x) = 4x^3 + 9x - 5$.

(i) Show that $(2x - 1)$ is a factor of f(x) and hence express f(x) as the product of a linear factor and a quadratic factor. **[4]**

(ii) (a) Show that the equation

$$4\sin 2\theta \cos 2\theta + \frac{5}{\cos 2\theta} = 13\tan 2\theta$$

can be expressed in the form

$$4\sin^3 2\theta + 9\sin 2\theta - 5 = 0.$$ **[4]**

(b) Hence solve the equation

$$4\sin 2\theta \cos 2\theta + \frac{5}{\cos 2\theta} = 13\tan 2\theta$$

for $0 \leqslant \theta \leqslant 360$ Give each answer in an exact form. **[4]**

Q11, (OCR 4752, Jun 2009, Q7)

Show that the equation $4\cos^2\theta = 4 - \sin\theta$ may be written in the form

$$4\sin^2\theta - \sin\theta = 0.$$

Hence solve the equation $4\cos^2\theta = 4 - \sin\theta$ for $0° \leqslant \theta \leqslant 180°$. **[5]**

Q12, (OCR 4752, Jun 2011, Q7)

Solve the equation $\tan\theta = 2\sin\theta$ for $0° \leqslant \theta \leqslant 360°$. **[4]**

Q13, (Jan 2008, Q9)

(i)

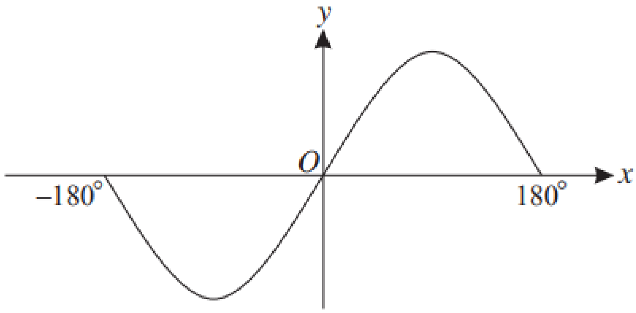

Fig. 1

Fig. 1 shows the curve $y = 2\sin x$ for values of x such that $-180° \leqslant x \leqslant 180°$. State the coordinates of the maximum and minimum points on this part of the curve. [2]

(ii)

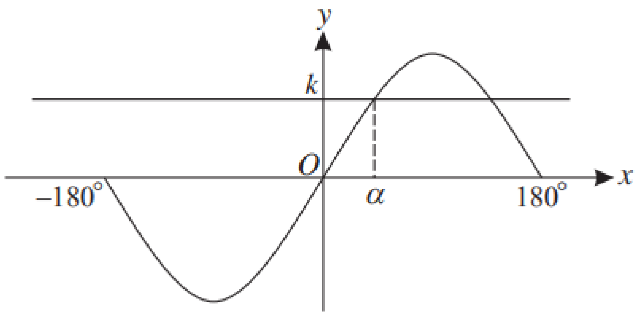

Fig. 2

Fig. 2 shows the curve $y = 2\sin x$ and the line $y = k$. The smallest positive solution of the equation $2\sin x = k$ is denoted by α. State, in terms of α, and in the range $-180° \leqslant x \leqslant 180°$,

(a) another solution of the equation $2\sin x = k$, [1]

(b) one solution of the equation $2\sin x = -k$. [1]

Q14, (OCR 4752, Jun 2013, Q9)

(i) Show that the equation $\dfrac{\tan\theta}{\cos\theta} = 1$ may be rewritten as $\sin\theta = 1 - \sin^2\theta$. **[2]**

(ii) Hence solve the equation $\dfrac{\tan\theta}{\cos\theta} = 1$ for $0° \leqslant \theta \leqslant 360°$. **[3]**

Q15, (Jun 2014, Q4)

(i) Show that the equation

$$\sin x - \cos x = \frac{6\cos x}{\tan x}$$

can be expressed in the form

$$\tan^2 x - \tan x - 6 = 0.$$ **[2]**

(ii) Hence solve the equation $\sin x - \cos x = \dfrac{6\cos x}{\tan x}$ for $0° \leqslant x \leqslant 360°$. **[4]**