Tangents and Normals to Curves (From OCR 4721)

Q1, (Specimen Paper, Q5)

(i) Solve the simultaneous equations

$$y = x^2 - 3x + 2, \qquad y = 3x - 7.$$ [5]

(ii) What can you deduce from the solution to part **(i)** about the graphs of $y = x^2 - 3x + 2$ and $y = 3x - 7$? [2]

(iii) Hence, or otherwise, find the equation of the normal to the curve $y = x^2 - 3x + 2$ at the point $(3, 2)$, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers. [4]

Q2, (Jun 2005, Q10)

(i) Given that $y = \frac{1}{3}x^3 - 9x$, find $\frac{dy}{dx}$. [2]

(ii) Find the coordinates of the stationary points on the curve $y = \frac{1}{3}x^3 - 9x$. [3]

(iii) Determine whether each stationary point is a maximum point or a minimum point. [3]

(iv) Given that $24x + 3y + 2 = 0$ is the equation of the tangent to the curve at the point (p, q), find p and q. [5]

Q3, (Jan 2009, Q10)

A curve has equation $y = x^2 + x$.

(i) Find the gradient of the curve at the point for which $x = 2$. **[2]**

(ii) Find the equation of the normal to the curve at the point for which $x = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[4]**

(iii) Find the values of k for which the line $y = kx - 4$ is a tangent to the curve. **[6]**

Q4, (Jan 2010, Q3)

Find the equation of the normal to the curve $y = x^3 - 4x^2 + 7$ at the point $(2, -1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[7]**

Q5, (Jan 2011, Q8)

(i) Find the equation of the tangent to the curve $y = 7 + 6x - x^2$ at the point P where $x = 5$, giving your answer in the form $ax + by + c = 0$. **[6]**

(ii) This tangent meets the x-axis at Q. Find the coordinates of the mid-point of PQ. **[3]**

(iii) Find the equation of the line of symmetry of the curve $y = 7 + 6x - x^2$. **[2]**

(iv) State the set of values of x for which $7 + 6x - x^2$ is an increasing function. **[2]**

Q6, (Jun 2011, Q10)

A curve has equation $y = (2x - 1)(x + 3)(x - 1)$.

(i) Sketch the curve, indicating the coordinates of all points of intersection with the axes. **[3]**

(ii) Show that the gradient of the curve at the point $P\,(1, 0)$ is 4. **[6]**

(iii) The line l is parallel to the tangent to the curve at the point P. The curve meets l at the point where $x = -2$. Find the equation of l, giving your answer in the form $y = mx + c$. **[4]**

(iv) Determine whether l is a tangent to the curve at the point where $x = -2$. **[3]**

Q7, (Jun 2012, Q6)

Find the equation of the normal to the curve $y = \frac{6}{x^2} - 5$ at the point on the curve where $x = 2$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[7]**

Q8, (Jun 2014, Q10)

A curve has equation $y = (x+2)^2(2x-3)$.

(i) Sketch the curve, giving the coordinates of all points of intersection with the axes. **[3]**

(ii) Find an equation of the tangent to the curve at the point where $x = -1$. Give your answer in the form $ax + by + c = 0$. **[9]**

Q9, (Jun 2015, Q7)

(a) Given that $f(x) = (x^2 + 3)(5 - x)$, find $f'(x)$. **[4]**

(b) Find the gradient of the curve $y = x^{-\frac{1}{3}}$ at the point where $x = -8$. **[4]**

Q10, (Jun 2017, Q11)

The normal to the curve $y = \frac{k}{x^2}$ at the point where $x = -3$ is parallel to the line $\frac{1}{2}y = 2 + 3x$.

(i) Determine the value of the constant k. **[6]**

(ii) Find the equation of the normal where $x = -3$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[4]**
