Stationary Points Exam Questions (From OCR 4721)

Note: All of these questions are from the old specification and are taken from a non-calculator papers. In all of these questions, in order to prepare you for questions that require “full working” or “detailed reasoning”, you should show all steps and keep calculator use to a minimum.

Q1, (Jun 2005, Q10)
(i) Given that \( y = \frac{1}{3}x^3 - 9x \), find \( \frac{dy}{dx} \). \([2]\]

(ii) Find the coordinates of the stationary points on the curve \( y = \frac{1}{3}x^3 - 9x \). \([3]\]

(iii) Determine whether each stationary point is a maximum point or a minimum point. \([3]\]

(iv) Given that \( 24x + 3y + 2 = 0 \) is the equation of the tangent to the curve at the point \((p, q)\), find \( p \) and \( q \). \([5]\]

Q2, (Jun 2006, Q6)
(i) Solve the equation \( x^4 - 10x^2 + 25 = 0 \). \([4]\]

(ii) Given that \( y = \frac{2}{3}x^5 - \frac{20}{3}x^3 + 50x + 3 \), find \( \frac{dy}{dx} \). \([2]\]

(iii) Hence find the number of stationary points on the curve \( y = \frac{2}{3}x^5 - \frac{20}{3}x^3 + 50x + 3 \). \([2]\]

Q3, (Jan 2007, Q8)
(i) Find the coordinates of the stationary points of the curve \( y = 27 + 9x - 3x^2 - x^3 \). \([6]\]

(ii) Determine, in each case, whether the stationary point is a maximum or minimum point. \([3]\]

(iii) Hence state the set of values of \( x \) for which \( 27 + 9x - 3x^2 - x^3 \) is an increasing function. \([2]\]

Q4, (Jan 2008, Q8)
(i) Find the coordinates of the stationary points on the curve \( y = x^3 + x^2 - x + 3 \). \([6]\]

(ii) Determine whether each stationary point is a maximum point or a minimum point. \([3]\]

(iii) For what values of \( x \) does \( x^3 + x^2 - x + 3 \) decrease as \( x \) increases? \([2]\]

Q5, (Jun 2008, Q8)
The curve \( y = x^3 - kx^2 + x - 3 \) has two stationary points.

(i) Find \( \frac{dy}{dx} \). \([2]\]

(ii) Given that there is a stationary point when \( x = 1 \), find the value of \( k \). \([3]\]

(iii) Determine whether this stationary point is a minimum or maximum point. \([2]\]

(iv) Find the \( x \)-coordinate of the other stationary point. \([3]\)
Q6, (Jan 2009, Q10)

The curve $y = x^3 + px^2 + 2$ has a stationary point when $x = 4$. Find the value of the constant $p$ and determine whether the stationary point is a maximum or minimum point. [7]

Q7, (Jun 2011, Q8)

(i) Find the coordinates of the stationary point on the curve $y = 3x^2 - \frac{6}{x} - 2$. [5]

(ii) Determine whether the stationary point is a maximum point or a minimum point. [2]

Q8, (Jun 2013, Q10)

The curve $y = (1 - x)(x^2 + 4x + k)$ has a stationary point when $x = -3$.

(i) Find the value of the constant $k$. [7]

(ii) Determine whether the stationary point is a maximum or minimum point. [2]

(iii) Given that $y = 9x - 9$ is the equation of the tangent to the curve at the point $A$, find the coordinates of $A$. [5]

Q9, (Jun 2014, Q8)

A curve has equation $y = 3x^3 - 7x + \frac{2}{x}$.

(i) Verify that the curve has a stationary point when $x = 1$. [5]

(ii) Determine the nature of this stationary point. [2]

(iii) The tangent to the curve at this stationary point meets the $y$-axis at the point $Q$. Find the coordinates of $Q$. [2]

Q10, (Jun 2015, Q9)

The curve $y = 2x^3 - ax^2 + 8x + 2$ passes through the point $B$ where $x = 4$.

(i) Given that $B$ is a stationary point of the curve, find the value of the constant $a$. [5]

(ii) Determine whether the stationary point $B$ is a maximum point or a minimum point. [2]

(iii) Find the $x$-coordinate of the other stationary point of the curve. [3]

Q11, (Jun 2016, Q11)

The curve $y = 4x^2 + \frac{a}{x} + 5$ has a stationary point. Find the value of the positive constant $a$ given that the $y$-coordinate of the stationary point is 32. [8]
The diagram shows part of the curve \( y = x^3 + px^2 + qx + r \). The curve passes through the point (3, 0) and there is a maximum point at \((-1, 0)\). Find the values of \( p \), \( q \) and \( r \) and hence determine the coordinates of the minimum point of the curve. [9]