Proof Exam Questions (From OCR 4751 unless otherwise stated)

Q1, (Jun 2006, Q4)

In each of the following cases choose one of the statements

$$P \Rightarrow Q \qquad P \Leftrightarrow Q \qquad P \Leftarrow Q$$

to describe the complete relationship between P and Q.

(i) P: $x^2 + x - 2 = 0$
Q: $x = 1$ [1]

(ii) P: $y^3 > 1$
Q: $y > 1$ [1]

Q2, (Jun 2007, Q3)

The converse of the statement ‘$P \Rightarrow Q$’ is ‘$Q \Rightarrow P$’.

Write down the converse of the following statement.

‘n is an odd integer $\Rightarrow 2n$ is an even integer.’

Show that this converse is false. [2]

Q3, (Jun 2011, Q10)

Factorise $n^3 + 3n^2 + 2n$. Hence prove that, when n is a positive integer, $n^3 + 3n^2 + 2n$ is always divisible by 6. **[3]**

Q4, (Jan 2012, Q9)

Complete each of the following by putting the best connecting symbol ($\Leftrightarrow$, $\Leftarrow$ or $\Rightarrow$) in the box. Explain your choice, giving full reasons.

(i) $n^3 + 1$ is an odd integer ☐ n is an even integer **[2]**

(ii) $(x-3)(x-2) > 0$ ☐ $x > 3$ **[2]**

Q5, (Jun 2013, Q9)

$n - 1$, n and $n + 1$ are any three consecutive integers.

(i) Show that the sum of these integers is always divisible by 3. **[1]**

(ii) Find the sum of the squares of these three consecutive integers and explain how this shows that the sum of the squares of any three consecutive integers is never divisible by 3. **[3]**

Q6, (Jun 2014, Q9)

You are given that n, $n + 1$ and $n + 2$ are three consecutive integers.

(i) Expand and simplify $n^2 + (n+1)^2 + (n+2)^2$. **[2]**

(ii) For what values of n will the sum of the squares of these three consecutive integers be an even number? Give a reason for your answer. **[2]**

Q7, (OCR 4753, Jun 2006, Q5)

Positive integers a, b and c are said to form a Pythagorean triple if $a^2 + b^2 = c^2$.

(i) Given that t is an integer greater than 1, show that $2t$, $t^2 - 1$ and $t^2 + 1$ form a Pythagorean triple. [3]

(ii) The two smallest integers of a Pythagorean triple are 20 and 21. Find the third integer.

Use this triple to show that not all Pythagorean triples can be expressed in the form $2t, t^2 - 1$ and $t^2 + 1$. [3]

Q8, (OCR 4753, Jun 2007, Q5)

Prove that the following statement is false.

For all integers n greater than or equal to 1, $n^2 + 3n + 1$ is a prime number. [2]

Q9, (OCR 4753, Jun 2009, Q7)

(i) Show that

(*A*) $(x - y)(x^2 + xy + y^2) = x^3 - y^3$,

(*B*) $(x + \frac{1}{2}y)^2 + \frac{3}{4}y^2 = x^2 + xy + y^2$. **[4]**

(ii) Hence prove that, for all real numbers x and y, if $x > y$ then $x^3 > y^3$. **[3]**

Q10, (OCR 4753, Jun 2011, Q7)

(i) Multiply out $(3^n + 1)(3^n - 1)$. **[1]**

(ii) Hence prove that if n is a positive integer then $3^{2n} - 1$ is divisible by 8. **[3]**

Q11, (OCR 4753, Jan 2013, Q7)

(i) Disprove the following statement:

$3^n + 2$ is prime for all integers $n \geqslant 0$. **[2]**

(ii) Prove that no number of the form 3^n (where n is a positive integer) has 5 as its final digit. **[2]**

Q12, (OCR 4753, Jan 2012, Q4)

Prove or disprove the following statement:

'No cube of an integer has 2 as its units digit.' **[2]**